Queen Mary
University of London

BSc/MSci Examination

Semester A Mid-Term Examination (Resit Paper)

SPA6311 Physical Cosmology Duration: 1 hour

YOU ARE NOT PERMITTED TO READ THE CONTENTS OF THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR.

Instructions:

Answer ALL questions from Section A. Answer ONLY TWO questions from Section B. Section A carries 20 marks, each question in section B carries 10 marks.

If you answer more questions than specified, only the first answers (up to the specified number) will be marked. Cross out any answers that you do not wish to be marked.

Only non-programmable calculators are permitted in this examination. Please state on your answer book the name and type of machine used.

Complete all rough workings in the answer book and cross through any work that is not to be assessed.

Important note: The academic regulations state that possession of unauthorised material at any time when a student is under examination conditions is an assessment offence and can lead to expulsion from QMUL.

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EXAM PAPERS MUST NOT BE REMOVED FROM THE EXAM ROOM.

Examiners:
Dr P. Bull
Dr D. Mulryne

SECTION A Answer ALL questions in Section A

Question A1

The FLRW line element can be written as

$$ds^2 = -c^2dt^2 + a^2\left(\frac{dr^2}{1-kr^2} + r^2d\theta^2 + r^2\sin^2\theta d\phi^2\right).$$

a) Briefly define the quantities a and k, stating whether they are constant or depend on time.

[3 marks]

b) Show that $da/dz = -a^2$, where z is the redshift.

[2 marks]

Question A2

The $\mathrm{C_{II}}$ (singly ionised) emission line of carbon has a rest-frame wavelength of $157.7\,\mu\mathrm{m}$.

a) A distant galaxy is observed to have a $\mathrm{C_{II}}$ emission line with wavelength $403.9\,\mu\mathrm{m}$. Calculate its redshift, z.

[2 marks]

b) Typical galaxies have peculiar velocities of several hundred km/s.

Assuming that the peculiar velocity of the distant galaxy is around this level ($< 1,000$ km/s), show that the Doppler shift contribution to its observed redshift is less than 1% of the total.

[3 marks]

Question A3

a) Write down how the densities of (i) matter, ρ_m, and (ii) the cosmological constant, ρ_Λ, depend on the scale factor, a.

[2 marks]

b) Derive an expression for the redshift at which the densities of matter and the cosmological constant are equal, $\rho_\Lambda(z) = \rho_m(z)$. Write your expression in terms of the fractional density parameters Ω_Λ and Ω_m.

[3 marks]

Question A4

Consider a universe with density parameters $\Omega_m = 0.9$, $\Omega_\Lambda = 0$, and $\Omega_r = 0$.

a) Calculate the value of Ω_k. Is this universe open, closed, or flat?

[3 marks]

b) Briefly describe the fate of this universe far into the future, paying particular attention to how its expansion rate will behave.

[2 marks]

SECTION B **Answer TWO questions from Section B**

Question B1

Consider a flat, radiation-only universe that is expanding.

a) By solving the Friedmann equation, find an expression for the scale factor as a function of time, $a(t)$, in this universe, subject to the initial condition $a = 0$ at $t = 0$.

[5 marks]

b) Derive an expression for the age of this universe as a function of a.

[2 marks]

c) The Hubble parameter in this universe is $H_0 = 84$ km/s/Mpc. Calculate the expansion rate, $H(z)$, at redshift $z = 0.8$.

[3 marks]

Question B2

The Raychaudhuri equation is given by

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} - \Lambda c^2 = -\frac{8\pi G}{c^2}p.$$

a) Use the Raychaudhuri equation to show that an expanding universe containing only pressureless matter and non-zero curvature (positive or negative) is always decelerating.

[5 marks]

b) Show that the conservation equation can be derived from the Friedmann and Raychaudhuri equations in a general FLRW universe.

(*Hint:* First multiply both sides of the Friedmann equation by a^2, and then take the time derivative of both sides of the resulting expression. Then, perform substitutions of the Friedmann and Raychaudhuri equations as necessary.)

[5 marks]

Question B3

Consider a flat universe that contains only a positive cosmological constant, Λ.

a) Derive an expression for the Hubble radius, $r_{\rm HR} = c/(aH)$, in this universe.

[4 marks]

b) Derive an expression for $r(a)$, the comoving distance travelled by light that was emitted at a scale factor a and that reaches the observer today (at $a = 1$), in this universe.

[6 marks]

Question B4

A Type Ia supernova explosion is observed in a galaxy at redshift $z = 1.42$. The flux of the supernova is measured to be $f = 3.17\,L_{\odot}/\mathrm{Mpc}^2$.

a) Based on measurements of its light curve, the luminosity of the supernova is inferred to be $L = 4.14 \times 10^9 L_{\odot}$. Use this information to calculate the luminosity distance to the galaxy, d_L, in Mpc.

[3 marks]

b) Assume that the physical diameter of the galaxy is 38 kpc. Calculate the angular size of the galaxy in arcsec.

[4 marks]

c) The Hubble parameter is measured to be $H_0 \approx 70$ km/s/Mpc. Show that neither the luminosity distance nor angular diameter distance satisfy the Hubble Law at this redshift. (Recall that the recession velocity is defined as $v_{\mathrm{rec}} \approx cz$.)

[3 marks]

End of Paper - An Appendix of 1 page follows

Useful information

The Friedmann equation is given by:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3},$$

where dots denote derivatives with respect to cosmic time, t.

The conservation equation for a cosmological fluid of density ρ and relativistic pressure p is

$$\dot{\rho} = -3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right).$$

The equation of state parameter, w, for a cosmological fluid is given by

$$p = w\rho c^2.$$

Unit conversions

$$\begin{aligned}
c &= 299792.6\,\mathrm{km/s} \\
G &= 6.674 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}} \\
1\,M_\odot &= 1.988 \times 10^{30}\,\mathrm{kg} \\
1\,L_\odot &= 3.828 \times 10^{26}\,\mathrm{W} \\
1\,\mathrm{pc} &= 3.0857 \times 10^{16}\,\mathrm{m} \\
1\,\mathrm{km/s/Mpc} &= 1.022 \times 10^{-12}\,\mathrm{yr^{-1}} \\
1\,\mathrm{deg} &= 3600\,\mathrm{arcsec}
\end{aligned}$$