Queen Mary
University of London

BSc/MSci Examination

Semester A Mid-Term Examination

SPA6311 Physical Cosmology **Duration: 1 hour**

YOU ARE NOT PERMITTED TO READ THE CONTENTS OF THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR.

Instructions:

Answer ALL questions from Section A. Answer ONLY TWO questions from Section B. Section A carries 20 marks, each question in section B carries 10 marks.

If you answer more questions than specified, only the first answers (up to the specified number) will be marked. Cross out any answers that you do not wish to be marked.

Only non-programmable calculators are permitted in this examination. Please state on your answer book the name and type of machine used.

Complete all rough workings in the answer book and cross through any work that is not to be assessed.

Important note: The academic regulations state that possession of unauthorised material at any time when a student is under examination conditions is an assessment offence and can lead to expulsion from QMUL.

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EXAM PAPERS MUST NOT BE REMOVED FROM THE EXAM ROOM.

Examiners:
Dr P. Bull
Dr D. Mulryne

SECTION A Answer ALL questions in Section A

Question A1

The Lyman-α ($n = 2 \rightarrow 1$) emission line of hydrogen is emitted at a wavelength of 121.6 nm in the rest frame.

a) A distant galaxy is observed to have a Lyman-α emission line with wavelength 121.9 nm. Calculate its redshift, z.

[2 marks]

b) The same galaxy is found to have a peculiar velocity of 450 km/s towards us. Calculate its true cosmological redshift (i.e. what its redshift would be if it had no peculiar motion).

[3 marks]

Question A2

a) Briefly define the terms (i) *homogeneity* and (ii) *isotropy*, in the context of a cosmological model.

[3 marks]

b) Write down the relation between the scale factor, a, and cosmological redshift, z, in an FLRW universe.

[2 marks]

Question A3

Consider a universe with an expansion rate of $H_0 = 65$ km/s/Mpc and a total density that is $1.2\times$ the critical density today.

a) Is this universe open, closed, or flat?

[2 marks]

b) Calculate the value of the curvature parameter, Ω_k, in this universe.

[3 marks]

Question A4

a) Write down how the densities of (i) matter, ρ_m, and (ii) radiation, ρ_r, depend on the scale factor, a.

[2 marks]

b) Derive an expression for the redshift at which the densities of matter and radiation are equal, $\rho_m(z) = \rho_r(z)$. Write your expression in terms of the fractional density parameters Ω_m and Ω_r.

[3 marks]

SECTION B **Answer TWO questions from Section B**

Question B1

Consider a flat, matter-only universe that is expanding.

a) By solving the Friedmann equation, find an expression for the scale factor as a function of time, $a(t)$, in this universe, subject to the initial condition $a = 0$ at $t = 0$.

[5 marks]

b) The expansion rate of this universe is measured to be 229 km/s/Mpc at a redshift of $z = 1$. Calculate the expansion rate today, H_0.

[3 marks]

c) Calculate the age of this universe at t_0 (today), in gigayears.

[2 marks]

Question B2

A spiral galaxy at redshift $z = 0.06$ is seen face-on, and is measured to subtend an angle of 22.4 arcsec. A Cepheid variable star is also observed in the galaxy, with a measured flux of $f = 2.65 \times 10^{-3}\, L_\odot/\mathrm{Mpc}^2$.

a) Based on measurements of its pulsation period, the Period-Luminosity relation predicts that the luminosity of the Cepheid should be $L = 2200 L_\odot$. Use this information to calculate the luminosity distance to the galaxy, d_L, in Mpc.

[3 marks]

b) Assuming that the observed redshift is the true cosmological redshift, calculate the physical diameter of the galaxy in kpc.

[4 marks]

c) The galaxy is later measured to have a peculiar velocity of 540 km/s towards us. Briefly explain how this will affect your calculation of its diameter.

[3 marks]

Question B3

The Raychaudhuri equation is given by

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} - \Lambda c^2 = -\frac{8\pi G}{c^2}p.$$

a) By taking the time derivative of the Friedmann equation, show that the Raychaudhuri equation can be derived from the Friedmann and conservation equations.

(*Hint:* First multiply both sides of the Friedmann equation by a^2, and then take the time derivative of both sides of the resulting expression. Then, perform substitutions of the Friedmann and conservation equations as necessary.)

[5 marks]

b) Consider a flat universe with only one type of matter/energy that has a constant equation of state parameter w.

Show that this universe will have accelerating expansion if $w < -\frac{1}{3}$.

[5 marks]

Question B4

The line element for a flat FLRW space-time is given by

$$ds^2 = -c^2dt^2 + a^2(t)\left(dr^2 + r^2d\theta^2 + r^2\sin^2\theta\, d\phi^2\right),$$

where $a(t)$ is the scale factor and (r, θ, ϕ) are spherical polar coordinates.

a) Use the line element to derive an expression for $r(a)$, the comoving distance travelled by light that was emitted at a scale factor a and that reaches the observer today (at $a = 1$).

Leave your expression in the form of an integral that is valid for any flat FLRW universe.

[5 marks]

b) Sketch a graph of the Hubble radius, $r_{\rm HR} \propto (aH)^{-1}$, as a function of the scale factor in (i) a matter-only universe, and (ii) a cosmological constant-only universe. Assume that both universes are flat and have the same Hubble parameter, H_0.

(Sketch both functions on the same axes, and label the lines clearly.)

[5 marks]

End of Paper - An Appendix of 1 page follows

Useful information

The Friedmann equation is given by:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3},$$

where k and Λ are constants, $a = a(t)$ is the scale factor, and dots denote derivatives with respect to cosmic time, t.
The conservation equation for a cosmological fluid of density ρ and relativistic pressure p is

$$\dot{\rho} = -3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right).$$

The equation of state parameter, w, for a cosmological fluid is given by

$$p = w\rho c^2.$$

Useful unit conversions:

$$\begin{aligned} 1\,M_\odot &= 1.988 \times 10^{30}\,\mathrm{kg} \\ 1\,L_\odot &= 3.828 \times 10^{26}\,\mathrm{W} \\ 1\,\mathrm{pc} &= 3.0857 \times 10^{16}\,\mathrm{m} \\ 1\,\mathrm{km/s/Mpc} &= 1.022 \times 10^{-12}\,\mathrm{yr}^{-1} \\ 1\,\mathrm{deg} &= 3600\,\mathrm{arcsec} \end{aligned}$$