



1. Consider the Q values for β -decay. For a nucleus (Z, A) derive formulas for the Q values in terms of *atomic* masses and *nuclear* masses for each type of β -decay.
2. A free neutron undergoes β^- decay. Write down this decay, and calculate the Q -value. What will the kinetic energy of the resulting proton be in the case where the momentum of the neutrino is negligible? [Hint: the electron is relativistic!]
3. In the lectures we sketched a diagram (called a Feynman diagram) for β^- decay, where a d quark was converted to a u quark. Try sketch one for β^+ (you can use google!).
4. The cross section for a neutrino from β -decay is $\sim 10^{-47} \text{ m}^2$. With this nominal cross section, some estimates of rates of interaction can be made. Multiplying the cross section times the density of a substance in atomic mass units gives a number of interactions per meter, and the inverse of that is an estimate of the mean free path. What is the mean free path of a neutrino in water of density 1000 kg/m^3 ? (Write your answer in light-years.) In a bathtub of water, what percentage of neutrinos would interact at all with the water?
5. Identify the important terms in the SEMF which govern spontaneous fission. Discuss the competing physical effects taking place. Why would a ball of neutrons be very unstable to fission?
6. Given that the activation energy of ^{236}U is 6.2 MeV what is the minimum energy α -particle which can induce fission when hitting a ^{232}Th target?
7. (Non-examinable material) On QM+ there is a resource paper on the derivation of the 'Bohr-Wheeler' spontaneous fission limit – we quoted simplified results in the lectures. Work through this calculation.
8. In the lectures we have shown that barrier penetration is more difficult for fission than it is for α -decay. Given that the Gamow factor is proportional to the integral $I = \int dr \sqrt{V(r) - Q}$, write down the tunnelling probability in terms of I . Consider a heavy nucleus with $A = 200$ that undergoes decay into two lighter nuclei with $A = 100$. How does this probability compare to the tunnelling probability of α -decay, assuming the value of I is the same in both cases? (You only need to provide a rough answer).

Some (potentially) useful information:

The radius of a nuclei may be approximated by $R \approx 1.2A^{1/3}$ fm.

The semi-empirical mass formula (SEMF) for the binding energy of a nucleon is

$$B(Z, A) = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(Z, A).$$

Constants in the SEMF: $a_V = 15.56, a_S = 17.23, a_C = 0.697, a_A = 23.28, a_P = 12.0$ where each number is in MeV.

Nuclear Shells: Protons

$$1s_{\frac{1}{2}} \downarrow_2 \quad 1p_{\frac{3}{2}} \downarrow_4 \quad 1p_{\frac{1}{2}} \downarrow_2 \quad 1d_{\frac{5}{2}} \downarrow_6 \quad 2s_{\frac{1}{2}} \downarrow_2 \quad 1d_{\frac{3}{2}} \downarrow_4 \quad 1f_{\frac{7}{2}} \downarrow_8 \quad 2p_{\frac{3}{2}} \downarrow_4 \quad 1f_{\frac{5}{2}} \downarrow_6 \quad 2p_{\frac{1}{2}} \downarrow_2 \quad 1g_{\frac{9}{2}} \downarrow_{10} \quad 1g_{\frac{7}{2}} \downarrow_8 \quad 2d_{\frac{5}{2}} \downarrow_6 \quad 1h_{\frac{11}{2}} \downarrow_{10} \quad 2d_{\frac{3}{2}} \downarrow_4 \quad 3s_{\frac{1}{2}} \downarrow_2 \quad 1h_{\frac{9}{2}} \downarrow_8 \quad 2f_{\frac{7}{2}} \downarrow_6 \quad \dots$$

Shells: Neutrons

$$1s_{\frac{1}{2}} \downarrow_2 \quad 1p_{\frac{3}{2}} \downarrow_4 \quad 1p_{\frac{1}{2}} \downarrow_2 \quad 1d_{\frac{5}{2}} \downarrow_6 \quad 2s_{\frac{1}{2}} \downarrow_2 \quad 1d_{\frac{3}{2}} \downarrow_4 \quad 1f_{\frac{7}{2}} \downarrow_8 \quad 2p_{\frac{3}{2}} \downarrow_4 \quad 1f_{\frac{5}{2}} \downarrow_6 \quad 2p_{\frac{1}{2}} \downarrow_2 \quad 1g_{\frac{9}{2}} \downarrow_{10} \quad 2d_{\frac{5}{2}} \downarrow_6 \quad 1g_{\frac{7}{2}} \downarrow_8 \quad 1h_{\frac{11}{2}} \downarrow_{10} \quad 2d_{\frac{3}{2}} \downarrow_4 \quad 3s_{\frac{1}{2}} \downarrow_2 \quad 2f_{\frac{7}{2}} \downarrow_6 \quad 1h_{\frac{9}{2}} \downarrow_8 \quad \dots$$

$\frac{e^2}{4\pi\epsilon_0}$	= 1.439965 MeV fm
Boltzmann's constant	$k_B = 8.6173303 \times 10^{-5}$ eV/K
Planck's constant	$h = 4.135668 \times 10^{-15}$ eV s
Speed of light	$c = 2.99792 \times 10^8$ m/s
Neutrino mean lifetime	881 s
Atomic mass unit	$1 u = 931.4940954 \text{ MeV}/c^2 = 1.66054 \times 10^{-27}$ kg
Mass of electron	$m_e = 5.4858 \times 10^{-4} u = 0.51099895 \text{ MeV}/c^2$
Mass of proton	$m_p = 1.00727646688 u = 938.27208 \text{ MeV}/c^2$
Mass of neutron	$m_n = 1.00866491578 u = 939.56541 \text{ MeV}/c^2$
Mass of ^1_1H	= 1.00782503 u
Mass of ^2_1H	= 2.01410178 u
Mass of ^3_1H	= 3.01604927 u
Mass of ^3_2He	= 3.01602932 u
Mass of ^4_2He	= 4.00260325 u
Mass of $^{232}_{90}\text{Th}$	= 232.038055 u
Mass of $^{234}_{90}\text{Th}$	= 234.043601 u
Mass of $^{233}_{92}\text{U}$	= 233.043930 u
Mass of $^{236}_{92}\text{U}$	= 236.045568 u
Mass of $^{238}_{92}\text{U}$	= 238.050788 u
Mass of $^{239}_{92}\text{U}$	= 239.054293 u
Mass of $^{240}_{94}\text{Pu}$	= 240.053811 u
Mass of $^{241}_{94}\text{Pu}$	= 241.056849 u
Mass of $^{242}_{94}\text{Pu}$	= 242.058741 u
Mass of the Sun	$M_{\odot} = 1.988 \times 10^{30}$ kg
Gravitational constant	$G = 6.67408 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$

Nuclei masses given are atomic masses.

You can look up other nuclear data from websites

<https://www-nds.iaea.org/relnsd/vcharthtml/VChartHTML.html>

<http://www.nndc.bnl.gov/nudat2/>

<http://atom.kaeri.re.kr/nuchart/>

<http://people.physics.anu.edu.au/~ecs103/chart/>