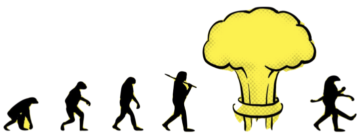
Queen Mary
University of London



SPA5302
EXERCISE SHEET 3

NUCLEAR PHYSICS & ASTROPHYSICS
2019

---

1. Discuss the following aspects of the potential which makes up the shell model when adding the spin-orbit term to the Woods-Saxon potential:
   (a) Why does the occupancy number of each energy level change when we add the spin-orbit term to the Woods-Saxon potential?
   (b) How many separate levels are there compared to the Woods-Saxon potential on its own?
   (c) Which levels split when adding the spin-orbit term?
   (d) List all the levels that are lifted above the next level in the Woods-Saxon potential.

Solution:

a) The potential now couples to the spin of the nucleon relative to the angular momentum of the state. This means that each spin state [relative to ℓ_z] has its own energy level.

b) Twice as many

c) all with $\ell \neq 0$

d) 1f5/2, 1g7/2, 2d3/2

2. Argue why the shell model can predict the state J^π for odd-A and e-e nuclei. For odd-odd nuclei, why can't we make such firm predictions?

Solution:

This follows from the assumption that if 2 n or 2 p occupy the same level then their total angular momentum couple to give zero. This implies that the spin and parity of a nucleon are the same as that of the last unpaired nucleon.

3. Sketch energy level diagrams using the shell model for the nuclei ^{21}Ne and ^{31}P, clearly labelling each energy levels. Identify the magic levels in each case. Give the ground state spin and parity for both cases.

Solution:

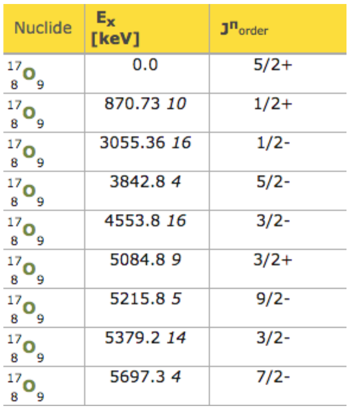
4. Give the spin and parity ground states for ^{7}Li, ^{29}Si, ^{85}Rb, (use the ordering of the shells overleaf). Use the notation $(n\ell_j)^k$ to say where the last unpaired nucleon is in each case.

Solution: ^{7}Li, ^{29}Si, ^{85}Rb,

$$3/2^{-}, 1/2^{+}, 3/2^{-}$$

5. Consider excited states of ^{17}O. List some different ways the energy can be elevated from its ground state by elevating one nucleon, and give the new spin and parity states the shell model would predict. In the figure overleaf, assume the proportional spacing of the energy levels is accurate. Try to order the excitation possibilities you have identified. Compare this to the real measured excitation states and compare the spin and parity states you have found. Does the model work well?

Solution:



Nuclide	E_x [keV]	J^{π}_{order}
$^{17}_{8}\mathrm{O}_{9}$	0.0	5/2+
$^{17}_{8}\mathrm{O}_{9}$	870.73 *10*	1/2+
$^{17}_{8}\mathrm{O}_{9}$	3055.36 *16*	1/2-
$^{17}_{8}\mathrm{O}_{9}$	3842.8 *4*	5/2-
$^{17}_{8}\mathrm{O}_{9}$	4553.8 *16*	3/2-
$^{17}_{8}\mathrm{O}_{9}$	5084.8 *9*	3/2+
$^{17}_{8}\mathrm{O}_{9}$	5215.8 *5*	9/2-
$^{17}_{8}\mathrm{O}_{9}$	5379.2 *14*	3/2-
$^{17}_{8}\mathrm{O}_{9}$	5697.3 *4*	7/2-

The ground state has the last unpaired neutron in the $1d_{5/2}$ level. The first excited state would be to jump that one up to the 2s level, giving a state of $1/2^{+}$ - which is correct. the next excitation might be to lift one n from the $1p_{1/2}$ level to join the lone one in the $1d_{5/2}$ level - the overall state would then be $1/2^{-}$, again correct. Thereafter things aren't so simple, and would require a jump to the $1f_{5/2}$ level, from the $1d_{5/2}$. Excited states then become a superposition of different possibilities . . .

6. By considering the binding energies of ^{15}O, ^{16}O and ^{17}O, estimate the separation of the $1p_{\frac{1}{2}}$ and $1d_{\frac{5}{2}}$ energy levels in MeV.

Solution:

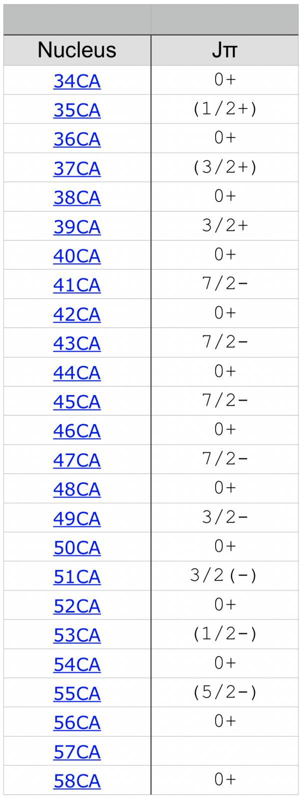
^{16}O fills the level $1p_{\frac{1}{2}}$, while ^{15}O has 1n missing from $1p_{\frac{1}{2}}$. So the binding energy of a n in $1p_{\frac{1}{2}}$ is $B(^{16}\mathrm{O}) - B(^{15}\mathrm{O}) = 15.66$MeV. Similarly, ^{17}O has 1n in the $1d_{\frac{5}{2}}$ energy level, meaning that the binding energy of 1 n in the $1d_{\frac{5}{2}}$ energy level is $B(^{17}\mathrm{O}) - B(^{16}\mathrm{O}) = 4.14$MeV. Consequently the separation of the $1p_{\frac{1}{2}}$ and $1d_{\frac{5}{2}}$ energy levels in MeV is the difference between these, which is 11.52MeV.

The binding energies of ^{15}O, ^{16}O and ^{17}O are 111.96, 127.62, 131.76 MeV respectively.

7. In the tables below nuclei with $Z = 20$ or $N = 20$ are given along with experimentally determined spin and parity states (where known – the brackets mean that the measurements are not fully verified).

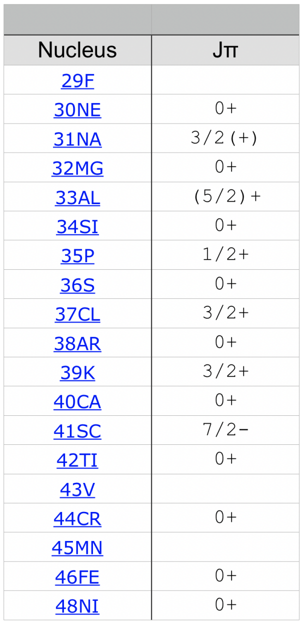
   Find the nuclei for which the shell model prediction is *incorrect*. (Be careful that you use the correct ordering of shells appropriate to neutrons or protons.) Speculate as to why this might be the case. Now look up a table of nuclides and see if you can find other cases where the shell model fails – does this fit with your speculation? (You can do a bit of research on this – it's a complicated problem!)

Nuclei with Z=20



Nucleus	Jπ
34CA	0+
35CA	(1/2+)
36CA	0+
37CA	(3/2+)
38CA	0+
39CA	3/2+
40CA	0+
41CA	7/2-
42CA	0+
43CA	7/2-
44CA	0+
45CA	7/2-
46CA	0+
47CA	7/2-
48CA	0+
49CA	3/2-
50CA	0+
51CA	3/2(-)
52CA	0+
53CA	(1/2-)
54CA	0+
55CA	(5/2-)
56CA	0+
57CA	
58CA	0+

Nuclei with N=20



Nucleus	Jπ
29F	
30NE	0+
31NA	3/2(+)
32MG	0+
33AL	(5/2)+
34SI	0+
35P	1/2+
36S	0+
37CL	3/2+
38AR	0+
39K	3/2+
40CA	0+
41SC	7/2-
42TI	0+
43V	
44CR	0+
45MN	
46FE	0+
48NI	0+

Data exported from http://www.nndc.bnl.gov/nudat2/indx_adopted.jsp

Some (potentially) useful information: The radius of a nuclei may be approximated by $R \approx 1.2A^{1/3}$ fm. The semi-empirical mass formula (SEMF) for the binding energy of a nucleon is

$$B(Z,A) = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(Z,A)\,.$$

Constants in the SEMF: $a_V = 15.56, a_S = 17.23, a_C = 0.697, a_A = 23.28, a_P = 12.0$ where each number is in MeV.

Nuclear Shells: Protons

$$1s_{\frac{1}{2}} \Big\downarrow_{2} 1p_{\frac{3}{2}} \mid 1p_{\frac{1}{2}} \Big\downarrow_{8} 1d_{\frac{5}{2}} \mid 2s_{\frac{1}{2}} \mid 1d_{\frac{3}{2}} \Big\downarrow_{20} 1f_{\frac{7}{2}} \Big\downarrow_{28} 2p_{\frac{3}{2}} \mid 1f_{\frac{5}{2}} \mid 2p_{\frac{1}{2}} \mid 1g_{\frac{9}{2}} \Big\downarrow_{50} 1g_{\frac{7}{2}} \mid 2d_{\frac{5}{2}} \mid 1h_{\frac{11}{2}} \mid 2d_{\frac{3}{2}} \mid 3s_{\frac{1}{2}} \Big\downarrow_{82} 1h_{\frac{9}{2}} \mid 2f_{\frac{7}{2}} \mid \cdots$$

Shells: Neutrons

$$1s_{\frac{1}{2}} \Big\downarrow_{2} 1p_{\frac{3}{2}} \mid 1p_{\frac{1}{2}} \Big\downarrow_{8} 1d_{\frac{5}{2}} \mid 2s_{\frac{1}{2}} \mid 1d_{\frac{3}{2}} \Big\downarrow_{20} 1f_{\frac{7}{2}} \Big\downarrow_{28} 2p_{\frac{3}{2}} \mid 1f_{\frac{5}{2}} \mid 2p_{\frac{1}{2}} \mid 1g_{\frac{9}{2}} \Big\downarrow_{50} 2d_{\frac{5}{2}} \mid 1g_{\frac{7}{2}} \mid 1h_{\frac{11}{2}} \mid 2d_{\frac{3}{2}} \mid 3s_{\frac{1}{2}} \Big\downarrow_{82} 2f_{\frac{7}{2}} \mid 1h_{\frac{9}{2}} \mid \cdots$$

$\dfrac{e^2}{4\pi\varepsilon_0}$	$= 1.439965\,\mathrm{MeV\,fm}$
Boltzmann's constant	$k_B = 8.6173303 \times 10^{-5}\,\mathrm{eV/K}$
Planck's constant	$h = 4.135668 \times 10^{-15}\,\mathrm{eV\,s}$
Speed of light	$c = 2.99792 \times 10^{8}\,\mathrm{m/s}$
Neutrino mean lifetime	881 s
Atomic mass unit	$1\,u = 931.4940954\,\mathrm{MeV}/c^2 = 1.66054 \times 10^{-27}\,\mathrm{kg}$
Mass of electron	$m_e = 5.4858 \times 10^{-4}\,u = 0.51099895\,\mathrm{MeV}/c^2$
Mass of proton	$m_p = 1.00727646688\,u = 938.27208\,\mathrm{MeV}/c^2$
Mass of neutron	$m_n = 1.00866491578\,u = 939.56541\,\mathrm{MeV}/c^2$
Mass of ${}^{1}_{1}\mathrm{H}$	$= 1.00782503\,u$
Mass of ${}^{2}_{1}\mathrm{H}$	$= 2.01410178\,u$
Mass of ${}^{3}_{1}\mathrm{H}$	$= 3.01604927\,u$
Mass of ${}^{3}_{2}\mathrm{He}$	$= 3.01602932\,u$
Mass of ${}^{4}_{2}\mathrm{He}$	$= 4.00260325\,u$
Mass of ${}^{232}_{90}\mathrm{Th}$	$= 232.038055\,u$
Mass of ${}^{234}_{90}\mathrm{Th}$	$= 234.043601\,u$
Mass of ${}^{235}_{92}\mathrm{U}$	$= 235.043930\,u$
Mass of ${}^{236}_{92}\mathrm{U}$	$= 236.045568\,u$
Mass of ${}^{238}_{92}\mathrm{U}$	$= 238.050788\,u$
Mass of ${}^{239}_{92}\mathrm{U}$	$= 239.054293\,u$
Mass of ${}^{240}_{94}\mathrm{Pu}$	$= 240.053811\,u$
Mass of ${}^{241}_{94}\mathrm{Pu}$	$= 241.056849\,u$
Mass of ${}^{242}_{94}\mathrm{Pu}$	$= 242.058741\,u$
Mass of the Sun	$M_\odot = 1.988 \times 10^{30}\,\mathrm{kg}$
Gravitational constant	$G = 6.67408 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$

Nuclei masses given are atomic masses.

You can look up other nuclear data from websites
`https://www-nds.iaea.org/relnsd/vcharthtml/VChartHTML.html`
`http://www.nndc.bnl.gov/nudat2/`
`http://atom.kaeri.re.kr/nuchart/`
`http://people.physics.anu.edu.au/~ecs103/chart/`