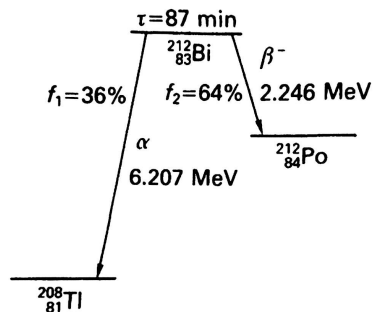


1.  $^{212}_{83}\text{Bi}$  can decay either by  $\alpha$ -particle emission (with change of energy  $Q = 6.2\text{MeV}$ ) or by  $\beta^-$  emission (with change of energy  $Q = 2.2\text{MeV}$ ), with branching fractions 36% and 64%. Sketch an energy level diagram showing this decay, labelling all the numbers given clearly. Label also the end products of the decays.



**Fig. 2.4** An example of a bimodal decay. The branching fractions  $f_1$  and  $f_2$  are given in per cent.

**Solution:** Scanned by CamScanner

2. A pure carbon archaeological item of mass 4g is excavated. An experimenter measures the activity and records 126 counts in a period of 1 hour, assumed to come from decays of  $^{14}\text{C}$  which has a decay constant of  $1.2092 \times 10^{-4}\text{yr}^{-1}$ . At the same time a background count of 0.010 Bq is also recorded. Assuming living matter has an abundance of  $1.0 \times 10^{-12}$  of  $^{14}\text{C}$ , what is the age of the artefact?

**Solution:**

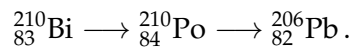
Activity of the sample = measured activity - background  $126/3600 - 0.01 = 0.025\text{ Bq}$

$^{14}\text{C}$  has  $\lambda = 1.2092 \times 10^{-4}\text{yr}^{-1}$  thus the mean lifetime  $\tau = 8269.9\text{yr}$  and Half-life =  $\ln 2/\lambda = 5732.2\text{yr}$

4g  $^{12}\text{C}$  contains  $4N_A/12$  atoms  $\sim 2 \times 10^{23}$  atoms

At time of death ( $t=0$ ) activity was due to one  $^{14}\text{C}$  in every  $10^{12}$  of  $^{12}\text{C}$  nuclei Thus nr  $^{14}\text{C}$  nuclei =  $10^{-12} \times 2 \times 10^{23} = 2 \times 10^{11}$  atoms of  $^{14}\text{C}$  Thus  $N_0 = 2 \times 10^{11}$  and  $N(t) = N_0 e^{-\lambda t}$ .  
 Activity  $A(t=0) = -dN/dt = \lambda N_0 = 2 \times 10^{11} \times 1.2092 \times 10^{-4} \text{yr}^{-1} = 24 \times 10^6 \text{yr}^{-1} / 31.5 \times 10^6 \text{s/yr} = 0.76 \text{Bq}$ .  
 Then,  $A(0)/A(T) = e^{\lambda T} \rightarrow \ln(0.76) - \ln(0.025) = \lambda T$ , giving age of the artefact  $T = 28.24 \times 10^3 \text{yr}$

3. An example of a decay chain is



What type of decay does each arrow correspond to? The mean lifetime for each decay is 7.2 days and 200 days respectively. If an initial sample is pure  ${}_{83}^{210}\text{Bi}$ , what are the relative amounts (as a percentage) of  ${}_{83}^{210}\text{Bi}$ ,  ${}_{84}^{210}\text{Po}$  and  ${}_{82}^{206}\text{Pb}$  after 1 month and 1 year?

**Solution:** First is  $\beta$  second is  $\alpha$ .

Let the first decay have  $\lambda_1 = 1/7.2 \text{ days}^{-1}$  and the second have  $\lambda_2 = 1/200 \text{ days}^{-1}$ . If we start with an initial sample  $N_0$  then after time  $t$  we have

$$N_1(t) = N_0 e^{-\lambda_1 t} \quad (1)$$

$$N_2(t) = N_0 \frac{\lambda_1}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t}). \quad (2)$$

at  $t = 30$  days we have

$$N_1(t) = N_0 e^{-30/7.2} \sim 0.015 N_0 \quad (3)$$

$$N_2(t) = N_0 \frac{1/7.2}{1/200 - 1/7.2} (e^{-30/7.2} - e^{-30/200}) \sim 0.877 N_0. \quad (4)$$

the first equation tells us theres 1.5% left of  ${}_{83}^{210}\text{Bi}$  after a year, and the second that  ${}_{84}^{210}\text{Po}$  makes up 87.7% of the final sample, the rest being  ${}_{82}^{206}\text{Pb}$ .

Now, at  $t = 365$  days we have

$$N_1(t) = N_0 e^{-365/7.2} \sim 10^{-23} N_0 \quad (5)$$

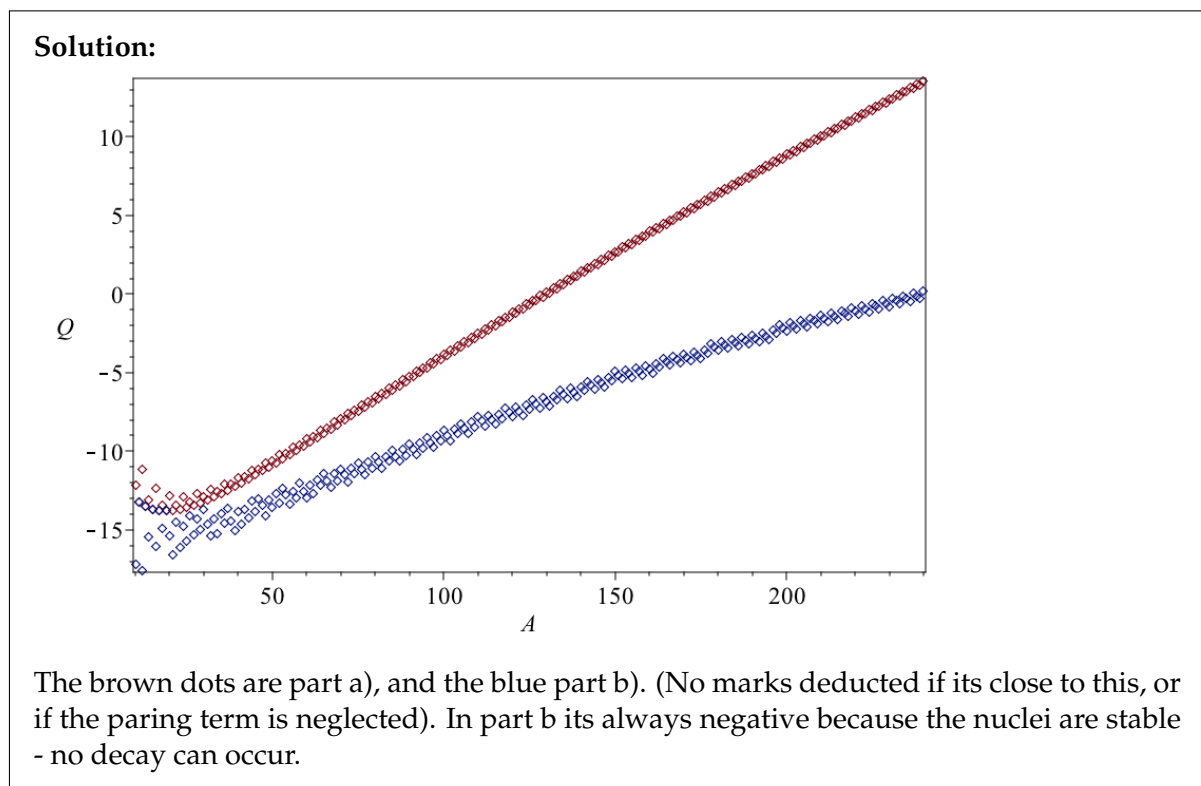
$$N_2(t) = N_0 \frac{1/7.2}{1/200 - 1/7.2} (e^{-365/7.2} - e^{-365/200}) \sim 0.167 N_0. \quad (6)$$

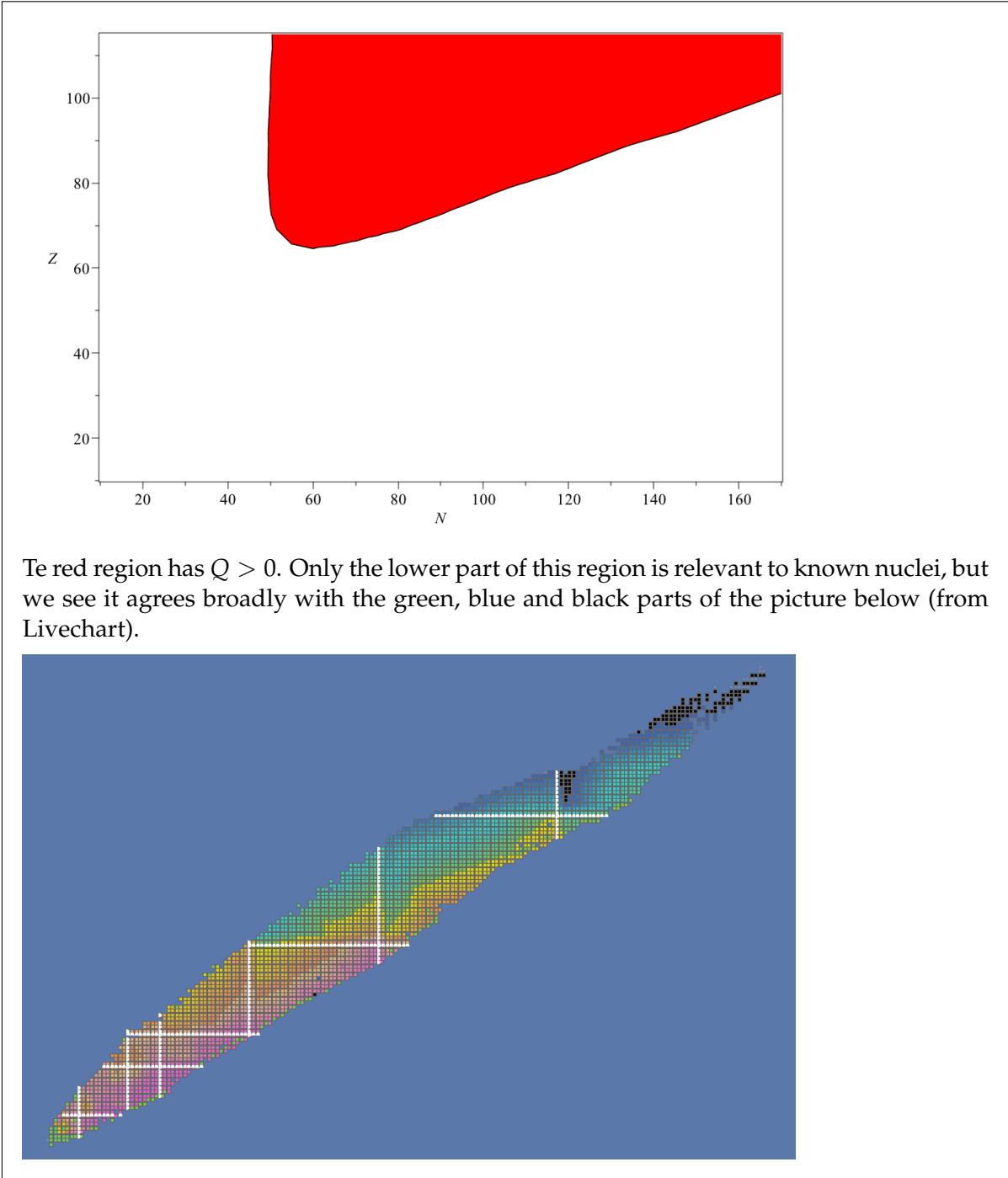
the first equation tells us theres basically nothing left of  ${}_{83}^{210}\text{Bi}$  after a year, and the second that  ${}_{84}^{210}\text{Po}$  makes up 16.7% of the final sample, the rest being  ${}_{82}^{206}\text{Pb}$ .

4. Consider  $\alpha$ -decay for a heavy nucleus. Sketch a suitable form for the potential well the  $\alpha$ -particle moves in, if modelled as fully formed inside the daughter nucleus. Sketch the form of the wavefunction for  $Q > 0$  and  $Q < 0$ .

**Solution:** Notes Fig 41. For  $Q < 0$  the particle is never free, and the wave function is exponentially damped for  $r > R$ .

5. In the notes and lectures we derived the Geiger-Nuttall Rule for  $\alpha$ -decay. Go through this calculation step by step making sure you understand the approximations used.
6. If you search around the internet for radioactive dating you will find a fair number of websites trying to debunk it as a flawed technique, usually for ideological reasons. Find the errors in their arguments.
7. (Requires some computing...) This question will explore which nuclei are most likely to undergo  $\alpha$ -decay. First have a look at the Livechart website given below. Colour the zones by 'Q alpha', and recall that  $Q > 0$  is necessary for  $\alpha$  decay. Note that the region with  $Q > 0$  has  $A > 100$  and lies above the valley of stability line – i.e., for proton-rich nuclei. We will explore this using the SEMF.
  - (a) Write down  $Q(Z, A)$  in terms of the binding energies of the parent and daughter nuclei, and use the SEMF to write this as a function of  $Z, A$ . Assuming  $Z = N$ , make a plot of this function between  $50 < A < 240$ . [You should find it crosses zero at  $A \sim 130$ ]
  - (b) Now plot the same but choose  $Z = Z_{min}$  where  $Z_{min}$  was found in Problem set 2. You should find this is always negative. Why?
  - (c) Finally plot  $Q$  as a contour plot (a 3d plot viewed from above) in the  $N, Z$  plane, and identify the region  $Q > 0$ . You should find it covers roughly the same region as on the live chart picture.





The red region has  $Q > 0$ . Only the lower part of this region is relevant to known nuclei, but we see it agrees broadly with the green, blue and black parts of the picture below (from Livechart).

*Some (potentially) useful information:*

The radius of a nuclei may be approximated by  $R \approx 1.2A^{1/3}$  fm.  
 The semi-empirical mass formula (SEMF) for the binding energy of a nucleon is

$$B(Z, A) = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(Z, A).$$

Constants in the SEMF:  $a_V = 15.56, a_S = 17.23, a_C = 0.697, a_A = 23.28, a_P = 12.0$  where each number is in MeV.

Nuclear Shells: Protons

$$1s_{\frac{1}{2}} \downarrow_2 \mid 1p_{\frac{3}{2}} \mid 1p_{\frac{1}{2}} \downarrow_8 \mid 1d_{\frac{5}{2}} \mid 2s_{\frac{1}{2}} \mid 1d_{\frac{3}{2}} \downarrow_{20} \mid 1f_{\frac{7}{2}} \downarrow_{28} \mid 2p_{\frac{3}{2}} \mid 1f_{\frac{5}{2}} \mid 2p_{\frac{1}{2}} \mid 1g_{\frac{9}{2}} \downarrow_{50} \mid 1g_{\frac{7}{2}} \mid 2d_{\frac{5}{2}} \mid 1h_{\frac{11}{2}} \mid 2d_{\frac{3}{2}} \mid 3s_{\frac{1}{2}} \downarrow_{82} \mid 1h_{\frac{9}{2}} \mid 2f_{\frac{7}{2}} \mid \dots$$

Shells: Neutrons

$$1s_{\frac{1}{2}} \downarrow_2 \mid 1p_{\frac{3}{2}} \mid 1p_{\frac{1}{2}} \downarrow_8 \mid 1d_{\frac{5}{2}} \mid 2s_{\frac{1}{2}} \mid 1d_{\frac{3}{2}} \downarrow_{20} \mid 1f_{\frac{7}{2}} \downarrow_{28} \mid 2p_{\frac{3}{2}} \mid 1f_{\frac{5}{2}} \mid 2p_{\frac{1}{2}} \mid 1g_{\frac{9}{2}} \downarrow_{50} \mid 2d_{\frac{5}{2}} \mid 1g_{\frac{7}{2}} \mid 1h_{\frac{11}{2}} \mid 2d_{\frac{3}{2}} \mid 3s_{\frac{1}{2}} \downarrow_{82} \mid 2f_{\frac{7}{2}} \mid 1h_{\frac{9}{2}} \mid \dots$$

$\frac{e^2}{4\pi\epsilon_0}$	= 1.439965 MeV fm
Boltzmann's constant	$k_B = 8.6173303 \times 10^{-5} \text{ eV/K}$
Planck's constant	$h = 4.135668 \times 10^{-15} \text{ eV s}$
Speed of light	$c = 2.99792 \times 10^8 \text{ m/s}$
Neutrino mean lifetime	881 s
Atomic mass unit	$1 u = 931.4940954 \text{ MeV}/c^2 = 1.66054 \times 10^{-27} \text{ kg}$
Mass of electron	$m_e = 5.4858 \times 10^{-4} u = 0.51099895 \text{ MeV}/c^2$
Mass of proton	$m_p = 1.00727646688 u = 938.27208 \text{ MeV}/c^2$
Mass of neutron	$m_n = 1.00866491578 u = 939.56541 \text{ MeV}/c^2$
Mass of $^1_1\text{H}$	= 1.00782503 u
Mass of $^2_1\text{H}$	= 2.01410178 u
Mass of $^3_1\text{H}$	= 3.01604927 u
Mass of $^3_2\text{He}$	= 3.01602932 u
Mass of $^4_2\text{He}$	= 4.00260325 u
Mass of $^{232}_{90}\text{Th}$	= 232.038055 u
Mass of $^{234}_{90}\text{Th}$	= 234.043601 u
Mass of $^{235}_{92}\text{U}$	= 235.043930 u
Mass of $^{236}_{92}\text{U}$	= 236.045568 u
Mass of $^{238}_{92}\text{U}$	= 238.050788 u
Mass of $^{239}_{92}\text{U}$	= 239.054293 u
Mass of $^{240}_{94}\text{Pu}$	= 240.053811 u
Mass of $^{241}_{94}\text{Pu}$	= 241.056849 u
Mass of $^{242}_{94}\text{Pu}$	= 242.058741 u
Mass of the Sun	$M_{\odot} = 1.988 \times 10^{30} \text{ kg}$
Gravitational constant	$G = 6.67408 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$

Nuclei masses given are atomic masses.

You can look up other nuclear data from websites

- <https://www-nds.iaea.org/relnsd/vcharthtml/VChartHTML.html>
- <http://www.nndc.bnl.gov/nudat2/>
- <http://atom.kaeri.re.kr/nuchart/>
- <http://people.physics.anu.edu.au/~ecs103/chart/>