Queen Mary
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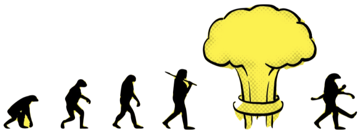

SPA5302 *NUCLEAR PHYSICS & ASTROPHYSICS*

EXERCISE SHEET 4 2019

1. ${}^{212}_{83}$Bi can decay either by α-particle emission (with change of energy $Q = 6.2$MeV) or by β^- emission (with change of energy $Q = 2.2$MeV), with branching fractions 36% and 64%. Sketch an energy level diagram showing this decay, labelling all the numbers given clearly. Label also the end products of the decays.

2. A pure carbon archaeological item of mass 4g is excavated. An experimenter measures the activity and records 126 counts in a period of 1 hour, assumed to come from decays of ^{14}C which has a decay constant of $1.2092 \times 10^{-4}\text{yr}^{-1}$. At the same time a background count of 0.010 Bq is also recorded. Assuming living matter has an abundance of 1.0×10^{-12} of ^{14}C, what is the age of the artefact?

3. An example of a decay chain is

$$ {}^{210}_{83}\text{Bi} \longrightarrow {}^{210}_{84}\text{Po} \longrightarrow {}^{206}_{82}\text{Pb}\,. $$

   What type of decay does each arrow correspond to? The mean lifetime for each decay is 7.2 days and 200 days respectively. If an initial sample is pure ${}^{210}_{83}$Bi, what are the relative amounts (as a percentage) of ${}^{210}_{83}$Bi, ${}^{210}_{84}$Po and ${}^{206}_{82}$Pb after 1 month and 1 year?

4. Consider α-decay for a heavy nucleus. Sketch a suitable form for the potential well the α-particle moves in, if modelled as fully formed inside the daughter nucleus. Sketch the form of the wavefunction for $Q > 0$ and $Q < 0$.

5. In the notes and lectures we derived the Geiger-Nuttall Rule for α-decay. Go through this calculation step by step making sure you understand the approximations used.

6. If you search around the internet for radioactive dating you will find a fair number of websites trying to debunk it as a flawed technique, usually for ideological reasons. Find the errors in their arguments.

7. (Requires some computing...) This question will explore which nuclei are most likely to undergo α-decay. First have a look at the Livechart website given below. Colour the zones by 'Q alpha', and recall that $Q > 0$ is necessary for α decay. Note that the region with $Q > 0$ has $A > 100$ and lies above the valley of stability line – i.e., for proton-rich nuclei. We will explore this using the SEMF.
   (a) Write down $Q(Z, A)$ in terms of the binding energies of the parent and daughter nuclei, and use the SEMF to write this as a function of Z, A. Assuming $Z = N$, make a plot of this function between $50 < A < 240$. [You should find it crosses zero at $A \sim 130$]
   (b) Now plot the same but choose $Z = Z_{min}$ where Z_{min} was found in Problem set 2. You should find this is always negative. Why?

(c) Finally plot Q as a contour plot (a 3d plot viewed from above) in the N, Z plane, and identify the region $Q > 0$. You should find it covers roughly the same region as on the live chart picture.

Some (potentially) useful information:

The radius of a nuclei may be approximated by $R \approx 1.2A^{1/3}$ fm.
The semi-empirical mass formula (SEMF) for the binding energy of a nucleon is

$$B(Z, A) = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(Z, A)\,.$$

Constants in the SEMF: $a_V = 15.56, a_S = 17.23, a_C = 0.697, a_A = 23.28, a_P = 12.0$ where each number is in MeV.

Nuclear Shells: Protons

$$1s_{\frac{1}{2}} \Big|_{\downarrow 2} 1p_{\frac{3}{2}} \,|\, 1p_{\frac{1}{2}} \Big|_{\downarrow 8} 1d_{\frac{5}{2}} \,|\, 2s_{\frac{1}{2}} \,|\, 1d_{\frac{3}{2}} \Big|_{\downarrow 20} 1f_{\frac{7}{2}} \Big|_{\downarrow 28} 2p_{\frac{3}{2}} \,|\, 1f_{\frac{5}{2}} \,|\, 2p_{\frac{1}{2}} \,|\, 1g_{\frac{9}{2}} \Big|_{\downarrow 50} 1g_{\frac{7}{2}} \,|\, 2d_{\frac{5}{2}} \,|\, 1h_{\frac{11}{2}} \,|\, 2d_{\frac{3}{2}} \,|\, 3s_{\frac{1}{2}} \Big|_{\downarrow 82} 1h_{\frac{9}{2}} \,|\, 2f_{\frac{7}{2}} \,|\, \cdots$$

Shells: Neutrons

$$1s_{\frac{1}{2}} \Big|_{\downarrow 2} 1p_{\frac{3}{2}} \,|\, 1p_{\frac{1}{2}} \Big|_{\downarrow 8} 1d_{\frac{5}{2}} \,|\, 2s_{\frac{1}{2}} \,|\, 1d_{\frac{3}{2}} \Big|_{\downarrow 20} 1f_{\frac{7}{2}} \Big|_{\downarrow 28} 2p_{\frac{3}{2}} \,|\, 1f_{\frac{5}{2}} \,|\, 2p_{\frac{1}{2}} \,|\, 1g_{\frac{9}{2}} \Big|_{\downarrow 50} 2d_{\frac{5}{2}} \,|\, 1g_{\frac{7}{2}} \,|\, 1h_{\frac{11}{2}} \,|\, 2d_{\frac{3}{2}} \,|\, 3s_{\frac{1}{2}} \Big|_{\downarrow 82} 2f_{\frac{7}{2}} \,|\, 1h_{\frac{9}{2}} \,|\, \cdots$$

$\frac{e^2}{4\pi\varepsilon_0}$	$= 1.439965$ MeV fm
Boltzmann's constant	$k_B = 8.6173303 \times 10^{-5}$ eV/K
Planck's constant	$h = 4.135668 \times 10^{-15}$ eV s
Speed of light	$c = 2.99792 \times 10^8$ m/s
Neutrino mean lifetime	881 s
Atomic mass unit	$1\,u = 931.4940954\,\text{MeV}/c^2 = 1.66054 \times 10^{-27}$ kg
Mass of electron	$m_e = 5.4858 \times 10^{-4}\,u = 0.51099895\,\text{MeV}/c^2$
Mass of proton	$m_p = 1.00727646688\,u = 938.27208\,\text{MeV}/c^2$
Mass of neutron	$m_n = 1.00866491578\,u = 939.56541\,\text{MeV}/c^2$
Mass of $^1_1$H	$= 1.00782503\,u$
Mass of $^2_1$H	$= 2.01410178\,u$
Mass of $^3_1$H	$= 3.01604927\,u$
Mass of $^3_2$He	$= 3.01602932\,u$
Mass of $^4_2$He	$= 4.00260325\,u$
Mass of $^{232}_{90}$Th	$= 232.038055\,u$
Mass of $^{234}_{90}$Th	$= 234.043601\,u$
Mass of $^{235}_{92}$U	$= 235.043930\,u$
Mass of $^{236}_{92}$U	$= 236.045568\,u$
Mass of $^{238}_{92}$U	$= 238.050788\,u$
Mass of $^{239}_{92}$U	$= 239.054293\,u$
Mass of $^{240}_{94}$Pu	$= 240.053811\,u$
Mass of $^{241}_{94}$Pu	$= 241.056849\,u$
Mass of $^{242}_{94}$Pu	$= 242.058741\,u$
Mass of the Sun	$M_\odot = 1.988 \times 10^{30}$ kg
Gravitational constant	$G = 6.67408 \times 10^{-11}\,\text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2}$

Nuclei masses given are atomic masses.

You can look up other nuclear data from websites
`https://www-nds.iaea.org/relnsd/vcharthtml/VChartHTML.html`
`http://www.nndc.bnl.gov/nudat2/`
`http://atom.kaeri.re.kr/nuchart/`
`http://people.physics.anu.edu.au/~ecs103/chart/`