

Week 1 - Practice Exam Question

(a) (i) When universe was half its present size.

$$a = 1 \text{ today}$$

$$a = 0.5 \text{ at } \frac{1}{2} \text{ present size}$$

$$\Rightarrow a = \frac{1}{1+z}, \text{ so } z = \frac{1}{a} - 1$$

$$z = \frac{1}{0.5} - 1 = 1$$

(ii) $\frac{1}{10}$ present size

$$\rightarrow a = 0.1, \text{ so } z = \frac{1}{0.1} - 1 = 9$$

(b) Emitted at $\lambda_{em} = 121.6 \text{ nm}$

Galaxy A observed at $f = 0.99 \times 10^{15} \text{ Hz}$
Galaxy B " " $f = 1.90 \times 10^{15} \text{ Hz}$

(i) Redshift of each galaxy

$$\lambda = c/f \quad \text{and} \quad 1+z = \frac{\lambda_{obs}}{\lambda_{em}}$$

$$= f_{em}/f_{obs}$$

$$f_{em} = \frac{3 \times 10^8 \text{ km/s}}{121.6 \times 10^{-9} \text{ m}} = 2.467 \times 10^{15} \text{ Hz}$$

$$Z = \frac{f_{em} - 1}{f_{obs}}$$

$$\rightarrow Z_A = \frac{2.467 \times 10^{15} \text{ Hz} - 1}{0.99 \times 10^{15} \text{ Hz}} = 1.49$$

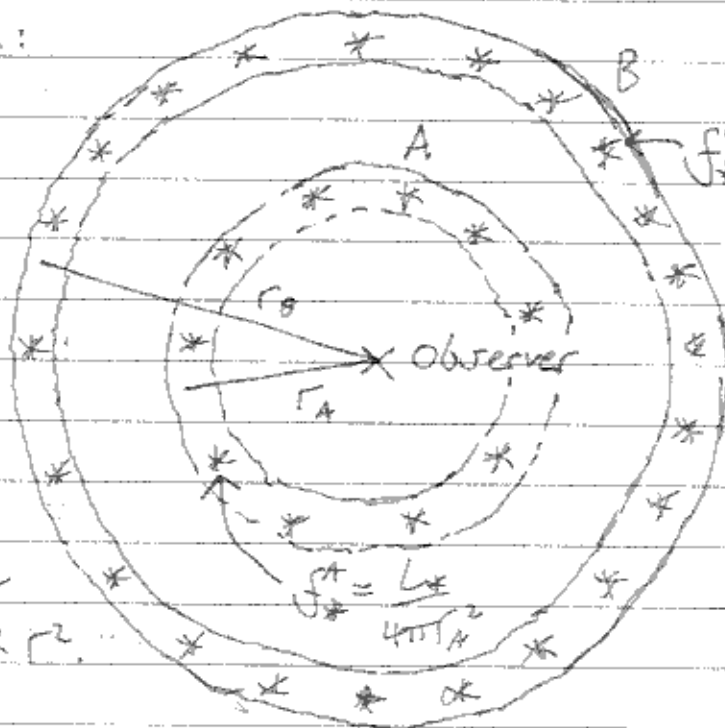
$$Z_B = \frac{2.467 \times 10^{15} \text{ Hz} - 1}{1.90 \times 10^{15} \text{ Hz}} = 0.298$$

(c) Olbers' Paradox

Important point is that flux of stars scales like $f \propto \frac{1}{r^2}$, but volume of shell (and therefore number of stars per shell) scales like $dV \propto N_* \propto r^2$.

So, the ~~total~~ flux received from each shell does not depend on distance!

Diagram:



No. of stars per shell $\propto r^2$

$$f_*^A = \frac{L_*}{4\pi r_A^2}$$

$$f_*^B = \frac{L_*}{4\pi r_B^2}$$

Number density of stars = const.

Flux per star $\propto \frac{1}{r^2}$

For an infinite, non-expanding universe, we would have to add up all shells to $r \rightarrow \infty$, so the total flux would $\rightarrow \infty$!

Expanding universe solves the paradox because light from very far away has not had chance to reach us yet - even if the universe was infinite, some parts of it are expanding away from us faster than c , so light couldn't reach us from there.

Another possible solution is that intervening gas and dust absorbs light from distant stars/galaxies. This does not solve the paradox though, as for an infinitely old universe, the gas/dust would absorb so much energy that it would heat up until it was ^{as} bright as the stars!

(d) Evidence for hot Big Bang model.

(A) Distant quasars.

(i) Very bright objects were observed using telescopes. They were found to have highly-redshifted spectra.

(ii) The abundance of these objects changed significantly with redshift. The abundance scaled as expected for a universe that was smaller/denser in the past. This implied that the universe had been expanding from a ~~the~~ denser (and so hotter) previous state.

(iii) If we assume that the expansion was adiabatic (i.e. didn't change the total amount of energy), this implies that the denser material in the past must also have been hotter on average.

(iv) This observation does not support the steady-state theory, which predicts that the average density of matter/galaxies/etc. should remain constant with time, as new matter is generated as space expands.

(B) Cosmic Microwave Background

(i) A background of blackbody radiation was observed using radio telescopes. The radiation was distributed isotropically and had a very low blackbody temperature.

(ii) In order for it to be so uniform/isotropic and so close to a blackbody, the radiation must have been emitted from everywhere in the universe during a previous epoch when it was much smoother/more homogeneous than it is today. This implies that the early universe is different to the late universe \Rightarrow cosmic evolution.

(iii) To generate such a perfect blackbody, the universe ~~was~~ must have been uniformly hot everywhere. The radiation ~~then~~ then cooled

as it redshifted (due to cosmic expansion).

(iv) If the CMB was being emitted at all times, it would look much less isotropic, since the universe immediately around us is quite homogeneous. The existence of the CMB implies that the universe was different in the past, which goes against the predictions of the Steady State theory.