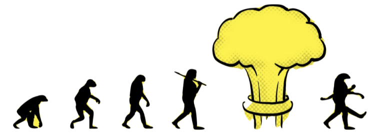

SPA5302 *NUCLEAR PHYSICS & ASTROPHYSICS 2019*

HOMEWORK SET 3 HAND IN: FRIDAY 1 NOVEMBER BY 4PM

Please write your answers clearly and neatly. **Include your working!** *Answers without proper reasoning will receive no marks. For some questions you will need to look up atomic masses, periodic tables etc.*

Hand in solutions to ALL questions. ONE of these will be marked and will count towards your class record.

1. Uranium-Lead dating can be used to date rocks that formed as early as the origin of the solar system, about 4.5 Gyr ago. The method relies on two separate decay chains, the uranium series from ^{238}U to ^{206}Pb, with a half-life of 4.47 billion years and the actinium series from ^{235}U to ^{207}Pb, with a half-life of 710 million years. Calculate the decay constants λ_{238} and λ_{235} for these two decays. [10]

   Assume a sample initially contains no lead, and that both decay chains undergo the usual radioactive decay law. Derive a formula for the ratio for ^{207}Pb/^{206}Pb in terms of the initial ratio of ^{235}U/^{238}U. If the initial ratio of ^{235}U/^{238}U=25% 3 billion years ago, what is the ratio ^{207}Pb/^{206}Pb today?

2. Consider the decay chain $1 \to 2 \to 3$, with decay constants λ_1 and λ_2, with species 3 a stable end product. Assuming that we start with an initial sample N_0 of 100% species 1, show that after time t the number of species 2 satisfies [10]

   $$N_2(t) = N_0 \frac{\lambda_1}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t}) \,. \qquad (1)$$

   Hence find a formula for $N_3(t)$. Make a sketch of the relative abundances for the 3 species over time.

3. Consider the α-decay [10]

   $$^{226}_{88}\mathrm{Ra} \longrightarrow {}^{222}_{86}\mathrm{Rn} + \alpha \,.$$

   Calculate the total energy released. By considering conservation of energy and momentum, derive a formula for the kinetic energy of the two decay products. Hence calculate the velocity in km/s of the $^{222}_{86}$Rn assuming the mother nucleus is initially at rest.

4. Consider isotopes of Uranium which undergo α-decay. Look up data for the half-life and Q-values for these isotopes. Compare to the form of the Geiger-Nuttal rule that we derived in lectures, [10]

   $$\log_{10} t_{\frac{1}{2}} \sim -22.3 + 0.14 \ln A - 1.4 A^{\frac{1}{6}} \sqrt{Z} + 1.72 \frac{Z}{\sqrt{Q}} MeV^{\frac{1}{2}} \,,$$

   for t in seconds) and identify the terms which are required to achieve a reasonable approximation. Make a sketch of the data and this formula with Q versus $\log_{10} t_{\frac{1}{2}}$.

Some (potentially) useful information:

The radius of a nuclei may be approximated by $R \approx 1.2A^{1/3}$ fm.
The semi-empirical mass formula (SEMF) for the binding energy of a nucleon is

$$B(Z,A) = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(Z,A)\,.$$

Constants in the SEMF: $a_V = 15.56, a_S = 17.23, a_C = 0.697, a_A = 23.28, a_P = 12.0$ where each number is in MeV.

Nuclear Shells: Protons

$$1s_{\frac{1}{2}} \Big\downarrow_2 1p_{\frac{3}{2}} \mid 1p_{\frac{1}{2}} \Big\downarrow_8 1d_{\frac{5}{2}} \mid 2s_{\frac{1}{2}} \mid 1d_{\frac{3}{2}} \Big\downarrow_{20} 1f_{\frac{7}{2}} \Big\downarrow_{28} 2p_{\frac{3}{2}} \mid 1f_{\frac{5}{2}} \mid 2p_{\frac{1}{2}} \mid 1g_{\frac{9}{2}} \Big\downarrow_{50} 1g_{\frac{7}{2}} \mid 2d_{\frac{5}{2}} \mid 1h_{\frac{11}{2}} \mid 2d_{\frac{3}{2}} \mid 3s_{\frac{1}{2}} \Big\downarrow_{82} 1h_{\frac{9}{2}} \mid 2f_{\frac{7}{2}} \mid \cdots$$

Shells: Neutrons

$$1s_{\frac{1}{2}} \Big\downarrow_2 1p_{\frac{3}{2}} \mid 1p_{\frac{1}{2}} \Big\downarrow_8 1d_{\frac{5}{2}} \mid 2s_{\frac{1}{2}} \mid 1d_{\frac{3}{2}} \Big\downarrow_{20} 1f_{\frac{7}{2}} \Big\downarrow_{28} 2p_{\frac{3}{2}} \mid 1f_{\frac{5}{2}} \mid 2p_{\frac{1}{2}} \mid 1g_{\frac{9}{2}} \Big\downarrow_{50} 2d_{\frac{5}{2}} \mid 1g_{\frac{7}{2}} \mid 1h_{\frac{11}{2}} \mid 2d_{\frac{3}{2}} \mid 3s_{\frac{1}{2}} \Big\downarrow_{82} 2f_{\frac{7}{2}} \mid 1h_{\frac{9}{2}} \mid \cdots$$

$\frac{e^2}{4\pi\varepsilon_0}$	$= 1.439965$ MeV fm
Boltzmann's constant	$k_B = 8.6173303 \times 10^{-5}$ eV/K
Planck's constant	$h = 4.135668 \times 10^{-15}$ eV s
Speed of light	$c = 2.99792 \times 10^8$ m/s
Neutrino mean lifetime	881 s
Atomic mass unit	$1\,u = 931.4940954$ MeV$/c^2 = 1.66054 \times 10^{-27}$ kg
Mass of electron	$m_e = 5.4858 \times 10^{-4}\,u = 0.51099895$ MeV$/c^2$
Mass of proton	$m_p = 1.00727646688\,u = 938.27208$ MeV$/c^2$
Mass of neutron	$m_n = 1.00866491578\,u = 939.56541$ MeV$/c^2$
Mass of $^1_1$H	$= 1.00782503\,u$
Mass of $^2_1$H	$= 2.01410178\,u$
Mass of $^3_1$H	$= 3.01604927\,u$
Mass of $^3_2$He	$= 3.01602932\,u$
Mass of $^4_2$He	$= 4.00260325\,u$
Mass of $^{232}_{90}$Th	$= 232.038055\,u$
Mass of $^{234}_{90}$Th	$= 234.043601\,u$
Mass of $^{235}_{92}$U	$= 235.043930\,u$
Mass of $^{236}_{92}$U	$= 236.045568\,u$
Mass of $^{238}_{92}$U	$= 238.050788\,u$
Mass of $^{239}_{92}$U	$= 239.054293\,u$
Mass of $^{240}_{94}$Pu	$= 240.053811\,u$
Mass of $^{241}_{94}$Pu	$= 241.056849\,u$
Mass of $^{242}_{94}$Pu	$= 242.058741\,u$
Mass of the Sun	$M_\odot = 1.988 \times 10^{30}$ kg
Gravitational constant	$G = 6.67408 \times 10^{-11}$ m^3 kg^{-1} s^{-2}

Nuclei masses given are atomic masses.

You can look up other nuclear data from websites
`https://www-nds.iaea.org/relnsd/vcharthtml/VChartHTML.html`
`http://www.nndc.bnl.gov/nudat2/`
`http://atom.kaeri.re.kr/nuchart/`
`http://people.physics.anu.edu.au/~ecs103/chart/`