Queen Mary
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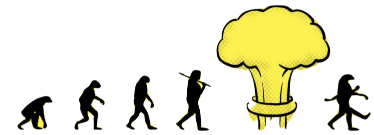

SPA5302 *NUCLEAR PHYSICS & ASTROPHYSICS*

EXERCISE SHEET 2 2019

Stared questions require using computing resources – this isn't a requirement to pass the course, but will help your understanding if you try them.

1. Use the SEMF to predict the *atomic* masses of ^{14}O, and ^{107}Ag, in atomic mass units. Compare these to the real values and estimate the percentage error in each case.

> **Solution:**
>
> ^{14}O: predicted: 14.00925965u, real: 14.008596u, error 0.0047%
>
> ^{107}Ag: predicted: 106.8974466u, real: 106.905091 u, error 0.0071%
>
> Very accurate!

2. (a) Starting from the SEMF for nuclear masses $M(Z, A)$, rearrange the formula to write it in the form
$$M(Z, A) = \alpha - \beta Z + \gamma Z^2 - \delta/c^2$$
Find expressions for the coefficients α, β and γ.

   (b) Considering the formula as a function of Z for A constant, and neglecting δ, find a formula for $Z(A)$ for the case when the mass is a minimum (ignoring the fact that Z, A are actually integers). Show that for small A this predicts $Z \simeq A/2$. What do you predict for large A?

   (c) * Using the formula for $Z(A)$ found above make a plot [using mathematica or an equivalent program] of $B(Z, A)/A$ for A up to 240. Comment on any discrepancies between the plot you have done and the plots given in the lecture slides.

   (d) * Make a 3d plot of $B(Z, A)/A$ as a function of N, Z and compare it to the table of nuclides. Discuss your findings.

> **Solution:**
>
> (Note the correction on the δ term from the version you worked from - this didn't affect the question.)
>
> a)
>
> $$\alpha = a_S A^{2/3} + A(a_A - a_V) + m_n, \;\; \beta = \frac{a_c}{A^{1/3}} + 4a_A - m_p + m_n, \;\; \gamma = \frac{a_c}{A^{1/3}} + 4\frac{a_A}{A}$$
>
> b)
>
> $$Z_{\text{min}} = \frac{4\, A^{4/3} a_A + a_C\, A}{8\, a_A\, A^{1/3} + 2\, a_C\, A}$$

For large A we have $2a_A A^{1/3}/a_C$

Figures shown in the lecture slides of week 3. The main difference is for low A, prediction is a bit high. At high A there are bumps in the real curve not contained in the SEMF.

3. For nuclei with $(N, Z) = (50, 28), (82, 50)$ (i.e., 'doubly magic' nuclei) calculate the binding energies in two ways: a) from the definition of B and by looking up atomic masses for the nuclei, and b) using the SEMF. What do you notice about your answers?

Solution:

$(N, Z) = (50, 28), (82, 50), (126, 82)$ has $B/A = 8.11, 8.275$ MeV. The actual values are: $8.23, 8.35$ MeV which are both significantly higher than the SEMF predicts.

4. Use the SEMF to predict the *nuclear* masses of ^{4}He, ^{14}C,^{196}Au, ^{244}Pu in atomic mass units. Compare these to the real values and estimate the percentage error in each case.

Solution: The (nuclear,atomic,real atomic, % error) predictions are:

$^4_2$He (4.006177186, 4.007274252 , 4.00260325413, 0.1166990e-0)

$^{14}_6$C (13.99957648, 14.00286768, 14.003241, 0.266596e-2)

^{196}Au (195.9175447, 195.9608788, 195.966569 0.00290366)

^{244}Pu (243.9973552, 244.0489173, 244.064205, 0.626381e-2)

5. In the notes we justified the asymmetry term using evenly spaced energy levels and the Pauli Exclusion Principle. Review this argument, and justify the cumulative effect sequence $1, 2, 5, 8, 13, 18, 25, \ldots$. Then show that the energy change must be $\propto (N - Z)^2$. [You can look up on wikipedia a derivation of this based on modelling the nucleus as a Fermi ball of protons and neutrons.]

Solution: This is covered in the notes.

6. Roughly sketch the distribution of mass estimated from the SEMF as a function of Z for $A = 125$ and $A = 128$ around their minima $Z \simeq 52$ and 54 respectively.

Solution: see Krane Fig 3.18

7. Consider isotopes of $_{56}$Ba, with $72 \le N \le 88$. Look up data for the neutron separation energy and plot this on a graph as a function of N. Describe the features you see with reference to the SEMF. What feature can't the SEMF explain?

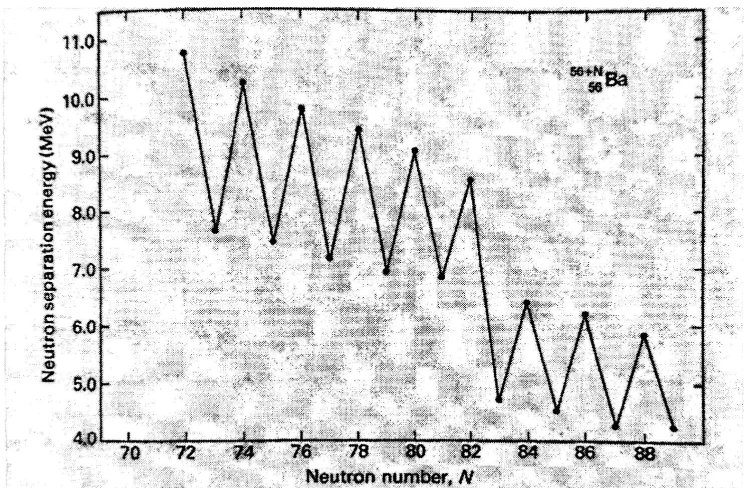

Fig. 4.4 The energy required to remove the last neutron (the **neutron separation energy**) from the isotopes of barium (${}_{56}$Ba). This energy is about 2 MeV greater when N is even than when N is odd. This 2 MeV represents the energy required to break the pairing favoured for like nucleons. The irregularity at $N = 82$ is one of the effects that we leave for discussion in Chapter 8. (Source of data: Wapstra and Audi 1985.)

Solution:

From Williams' book.

Some (potentially) useful information: The radius of a nuclei may be approximated by $R \approx 1.2A^{1/3}$ fm. The semi-empirical mass formula (SEMF) for the binding energy of a nucleon is

$$B(Z,A) = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(Z,A)\,.$$

Constants in the SEMF: $a_V = 15.56, a_S = 17.23, a_C = 0.697, a_A = 23.28, a_P = 12.0$ where each number is in MeV.

Nuclear Shells: Protons

$$1s_{\frac{1}{2}}\Big\downarrow_{2} 1p_{\frac{3}{2}} \mid 1p_{\frac{1}{2}}\Big\downarrow_{8} 1d_{\frac{5}{2}} \mid 2s_{\frac{1}{2}} \mid 1d_{\frac{3}{2}}\Big\downarrow_{20} 1f_{\frac{7}{2}}\Big\downarrow_{28} 2p_{\frac{3}{2}} \mid 1f_{\frac{5}{2}} \mid 2p_{\frac{1}{2}} \mid 1g_{\frac{9}{2}}\Big\downarrow_{50} 1g_{\frac{7}{2}} \mid 2d_{\frac{5}{2}} \mid 1h_{\frac{11}{2}} \mid 2d_{\frac{3}{2}} \mid 3s_{\frac{1}{2}}\Big\downarrow_{82} 1h_{\frac{9}{2}} \mid 2f_{\frac{7}{2}} \mid \cdots$$

Shells: Neutrons

$$1s_{\frac{1}{2}}\Big\downarrow_{2} 1p_{\frac{3}{2}} \mid 1p_{\frac{1}{2}}\Big\downarrow_{8} 1d_{\frac{5}{2}} \mid 2s_{\frac{1}{2}} \mid 1d_{\frac{3}{2}}\Big\downarrow_{20} 1f_{\frac{7}{2}}\Big\downarrow_{28} 2p_{\frac{3}{2}} \mid 1f_{\frac{5}{2}} \mid 2p_{\frac{1}{2}} \mid 1g_{\frac{9}{2}}\Big\downarrow_{50} 2d_{\frac{5}{2}} \mid 1g_{\frac{7}{2}} \mid 1h_{\frac{11}{2}} \mid 2d_{\frac{3}{2}} \mid 3s_{\frac{1}{2}}\Big\downarrow_{82} 2f_{\frac{7}{2}} \mid 1h_{\frac{9}{2}} \mid \cdots$$

$\frac{e^2}{4\pi\varepsilon_0}$	$= 1.439965\,\mathrm{MeV\,fm}$
Boltzmann's constant	$k_B = 8.6173303 \times 10^{-5}\,\mathrm{eV/K}$
Planck's constant	$h = 4.135668 \times 10^{-15}\,\mathrm{eV\,s}$
Speed of light	$c = 2.99792 \times 10^{8}\,\mathrm{m/s}$
Neutrino mean lifetime	881 s
Atomic mass unit	$1\,u = 931.4940954\,\mathrm{MeV}/c^2 = 1.66054 \times 10^{-27}\,\mathrm{kg}$
Mass of electron	$m_e = 5.4858 \times 10^{-4}\,u = 0.51099895\,\mathrm{MeV}/c^2$
Mass of proton	$m_p = 1.00727646688\,u = 938.27208\,\mathrm{MeV}/c^2$
Mass of neutron	$m_n = 1.00866491578\,u = 939.56541\,\mathrm{MeV}/c^2$
Mass of $^{1}_{1}\mathrm{H}$	$= 1.00782503\,u$
Mass of $^{2}_{1}\mathrm{H}$	$= 2.01410178\,u$
Mass of $^{3}_{1}\mathrm{H}$	$= 3.01604927\,u$
Mass of $^{3}_{2}\mathrm{He}$	$= 3.01602932\,u$
Mass of $^{4}_{2}\mathrm{He}$	$= 4.00260325\,u$
Mass of $^{232}_{90}\mathrm{Th}$	$= 232.038055\,u$
Mass of $^{234}_{90}\mathrm{Th}$	$= 234.043601\,u$
Mass of $^{235}_{92}\mathrm{U}$	$= 235.043930\,u$
Mass of $^{236}_{92}\mathrm{U}$	$= 236.045568\,u$
Mass of $^{238}_{92}\mathrm{U}$	$= 238.050788\,u$
Mass of $^{239}_{92}\mathrm{U}$	$= 239.054293\,u$
Mass of $^{240}_{94}\mathrm{Pu}$	$= 240.053811\,u$
Mass of $^{241}_{94}\mathrm{Pu}$	$= 241.056849\,u$
Mass of $^{242}_{94}\mathrm{Pu}$	$= 242.058741\,u$
Mass of the Sun	$M_\odot = 1.988 \times 10^{30}\,\mathrm{kg}$
Gravitational constant	$G = 6.67408 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$

Nuclei masses given are atomic masses.

You can look up other nuclear data from websites
`https://www-nds.iaea.org/relnsd/vcharthtml/VChartHTML.html`
`http://www.nndc.bnl.gov/nudat2/`
`http://atom.kaeri.re.kr/nuchart/`
`http://people.physics.anu.edu.au/~ecs103/chart/`