



1. Discuss the following aspects of the potential which makes up the shell model when adding the spin-orbit term to the Woods-Saxon potential:
  - (a) Why does the occupancy number of each energy level change when we add the spin-orbit term to the Woods-Saxon potential?
  - (b) How many separate levels are there compared to the Woods-Saxon potential on its own?
  - (c) Which levels split when adding the spin-orbit term?
  - (d) List all the levels that are lifted above the next level in the Woods-Saxon potential.
2. Argue why the shell model can predict the state  $J^\pi$  for odd- $A$  and e-e nuclei. For odd-odd nuclei, why can't we make such firm predictions?
3. Sketch energy level diagrams using the shell model for the nuclei  $^{21}\text{Ne}$  and  $^{31}\text{P}$ , clearly labelling each energy levels. Identify the magic levels in each case. Give the ground state spin and parity for both cases.
4. Give the spin and parity ground states for  $^7\text{Li}$ ,  $^{29}\text{Si}$ ,  $^{85}\text{Rb}$ , (use the ordering of the shells overleaf). Use the notation  $(nl_j)^k$  to say where the last unpaired nucleon is in each case.
5. Consider excited states of  $^{17}\text{O}$ . List some different ways the energy can be elevated from its ground state by elevating one nucleon, and give the new spin and parity states the shell model would predict. In the figure overleaf, assume the proportional spacing of the energy levels is accurate. Try to order the excitation possibilities you have identified. Compare this to the real measured excitation states and compare the spin and parity states you have found. Does the model work well?
6. By considering the binding energies of  $^{15}\text{O}$ ,  $^{16}\text{O}$  and  $^{17}\text{O}$ , estimate the separation of the  $1p_{\frac{1}{2}}$  and  $1d_{\frac{5}{2}}$  energy levels in MeV.

7. In the tables below nuclei with  $Z = 20$  or  $N = 20$  are given along with experimentally determined spin and parity states (where known – the brackets mean that the measurements are not fully verified).

Find the nuclei for which the shell model prediction is *incorrect*. (Be careful that you use the correct ordering of shells appropriate to neutrons or protons.) Speculate as to why this might be the case. Now look up a table of nuclides and see if you can find other cases where the shell model fails – does this fit with your speculation? (You can do a bit of research on this – it's a complicated problem!)

Nuclei with Z=20

| Nucleus              | J $\pi$ |
|----------------------|---------|
| <a href="#">34CA</a> | 0+      |
| <a href="#">35CA</a> | (1/2+)  |
| <a href="#">36CA</a> | 0+      |
| <a href="#">37CA</a> | (3/2+)  |
| <a href="#">38CA</a> | 0+      |
| <a href="#">39CA</a> | 3/2+    |
| <a href="#">40CA</a> | 0+      |
| <a href="#">41CA</a> | 7/2-    |
| <a href="#">42CA</a> | 0+      |
| <a href="#">43CA</a> | 7/2-    |
| <a href="#">44CA</a> | 0+      |
| <a href="#">45CA</a> | 7/2-    |
| <a href="#">46CA</a> | 0+      |
| <a href="#">47CA</a> | 7/2-    |
| <a href="#">48CA</a> | 0+      |
| <a href="#">49CA</a> | 3/2-    |
| <a href="#">50CA</a> | 0+      |
| <a href="#">51CA</a> | 3/2(-)  |
| <a href="#">52CA</a> | 0+      |
| <a href="#">53CA</a> | (1/2-)  |
| <a href="#">54CA</a> | 0+      |
| <a href="#">55CA</a> | (5/2-)  |
| <a href="#">56CA</a> | 0+      |
| <a href="#">57CA</a> |         |
| <a href="#">58CA</a> | 0+      |

Nuclei with N=20

| Nucleus              | J $\pi$ |
|----------------------|---------|
| <a href="#">29F</a>  |         |
| <a href="#">30NE</a> | 0+      |
| <a href="#">31NA</a> | 3/2(+)  |
| <a href="#">32MG</a> | 0+      |
| <a href="#">33AL</a> | (5/2)+  |
| <a href="#">34SI</a> | 0+      |
| <a href="#">35P</a>  | 1/2+    |
| <a href="#">36S</a>  | 0+      |
| <a href="#">37CL</a> | 3/2+    |
| <a href="#">38AR</a> | 0+      |
| <a href="#">39K</a>  | 3/2+    |
| <a href="#">40CA</a> | 0+      |
| <a href="#">41SC</a> | 7/2-    |
| <a href="#">42TI</a> | 0+      |
| <a href="#">43V</a>  |         |
| <a href="#">44CR</a> | 0+      |
| <a href="#">45MN</a> |         |
| <a href="#">46FE</a> | 0+      |
| <a href="#">48NI</a> | 0+      |

Data exported from [http://www.nndc.bnl.gov/nudat2/indx\\_adopted.jsp](http://www.nndc.bnl.gov/nudat2/indx_adopted.jsp)

**Some (potentially) useful information:** The radius of a nuclei may be approximated by  $R \approx 1.2A^{1/3}$  fm. The semi-empirical mass formula (SEMF) for the binding energy of a nucleon is

$$B(Z, A) = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(Z, A).$$

Constants in the SEMF:  $a_V = 15.56, a_S = 17.23, a_C = 0.697, a_A = 23.28, a_P = 12.0$  where each number is in MeV.

Nuclear Shells: Protons

$$1s_{\frac{1}{2}} \downarrow_2 \quad 1p_{\frac{3}{2}} \downarrow_4 \quad 1p_{\frac{1}{2}} \downarrow_2 \quad 1d_{\frac{5}{2}} \downarrow_6 \quad 2s_{\frac{1}{2}} \downarrow_2 \quad 1d_{\frac{3}{2}} \downarrow_{10} \quad 1f_{\frac{7}{2}} \downarrow_{14} \quad 2p_{\frac{3}{2}} \downarrow_{10} \quad 1f_{\frac{5}{2}} \downarrow_{10} \quad 2p_{\frac{1}{2}} \downarrow_2 \quad 1g_{\frac{9}{2}} \downarrow_{14} \quad 1g_{\frac{7}{2}} \downarrow_{10} \quad 2d_{\frac{5}{2}} \downarrow_{10} \quad 1h_{\frac{11}{2}} \downarrow_{12} \quad 2d_{\frac{3}{2}} \downarrow_{10} \quad 3s_{\frac{1}{2}} \downarrow_2 \quad 1h_{\frac{9}{2}} \downarrow_{10} \quad 2f_{\frac{7}{2}} \downarrow_{14} \quad \dots$$

Shells: Neutrons

$$1s_{\frac{1}{2}} \downarrow_2 \quad 1p_{\frac{3}{2}} \downarrow_4 \quad 1p_{\frac{1}{2}} \downarrow_2 \quad 1d_{\frac{5}{2}} \downarrow_6 \quad 2s_{\frac{1}{2}} \downarrow_2 \quad 1d_{\frac{3}{2}} \downarrow_{10} \quad 1f_{\frac{7}{2}} \downarrow_{14} \quad 2p_{\frac{3}{2}} \downarrow_{10} \quad 1f_{\frac{5}{2}} \downarrow_{10} \quad 2p_{\frac{1}{2}} \downarrow_2 \quad 1g_{\frac{9}{2}} \downarrow_{14} \quad 2d_{\frac{5}{2}} \downarrow_{10} \quad 1g_{\frac{7}{2}} \downarrow_{10} \quad 1h_{\frac{11}{2}} \downarrow_{12} \quad 2d_{\frac{3}{2}} \downarrow_{10} \quad 3s_{\frac{1}{2}} \downarrow_2 \quad 2f_{\frac{7}{2}} \downarrow_{14} \quad 1h_{\frac{9}{2}} \downarrow_{10} \quad \dots$$

|                                |  |
|--------------------------------|--|
| $\frac{e^2}{4\pi\epsilon_0}$   | = 1.439965 MeV fm  |
| Boltzmann's constant           | $k_B = 8.6173303 \times 10^{-5}$ eV/K                                    |
| Planck's constant              | $h = 4.135668 \times 10^{-15}$ eV s                                      |
| Speed of light                 | $c = 2.99792 \times 10^8$ m/s  |
| Neutrino mean lifetime         | 881 s  |
| Atomic mass unit               | $1 u = 931.4940954 \text{ MeV}/c^2 = 1.66054 \times 10^{-27}$ kg         |
| Mass of electron               | $m_e = 5.4858 \times 10^{-4} u = 0.51099895 \text{ MeV}/c^2$             |
| Mass of proton                 | $m_p = 1.00727646688 u = 938.27208 \text{ MeV}/c^2$                      |
| Mass of neutron                | $m_n = 1.00866491578 u = 939.56541 \text{ MeV}/c^2$                      |
| Mass of $^1_1\text{H}$         | = 1.00782503 u   |
| Mass of $^2_1\text{H}$         | = 2.01410178 u   |
| Mass of $^3_1\text{H}$         | = 3.01604927 u   |
| Mass of $^3_2\text{He}$        | = 3.01602932 u   |
| Mass of $^4_2\text{He}$        | = 4.00260325 u   |
| Mass of $^{232}_{90}\text{Th}$ | = 232.038055 u   |
| Mass of $^{234}_{90}\text{Th}$ | = 234.043601 u   |
| Mass of $^{235}_{92}\text{U}$  | = 235.043930 u   |
| Mass of $^{236}_{92}\text{U}$  | = 236.045568 u   |
| Mass of $^{238}_{92}\text{U}$  | = 238.050788 u   |
| Mass of $^{239}_{92}\text{U}$  | = 239.054293 u   |
| Mass of $^{240}_{94}\text{Pu}$ | = 240.053811 u   |
| Mass of $^{241}_{94}\text{Pu}$ | = 241.056849 u   |
| Mass of $^{242}_{94}\text{Pu}$ | = 242.058741 u   |
| Mass of the Sun                | $M_{\odot} = 1.988 \times 10^{30}$ kg                                    |
| Gravitational constant         | $G = 6.67408 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ |

Nuclei masses given are atomic masses.

You can look up other nuclear data from websites

<https://www-nds.iaea.org/relnsd/vcharthtml/VChartHTML.html>

<http://www.nndc.bnl.gov/nudat2/>

<http://atom.kaeri.re.kr/nuchart/>

<http://people.physics.anu.edu.au/~ecs103/chart/>