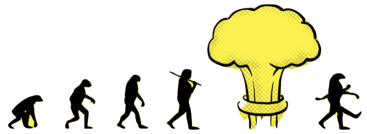

SPA5302

EXERCISE SHEET 2

NUCLEAR PHYSICS & ASTROPHYSICS

2019

Stared questions require using computing resources – this isn't a requirement to pass the course, but will help your understanding if you try them.

1. Use the SEMF to predict the *atomic* masses of ^{14}O, and ^{107}Ag, in atomic mass units. Compare these to the real values and estimate the percentage error in each case.
2. (a) Starting from the SEMF for nuclear masses $M(Z, A)$, rearrange the formula to write it in the form
   $$M(Z, A) = \alpha - \beta Z + \gamma Z^2 - \delta / c^2$$
   Find expressions for the coefficients α, β and γ.

   (b) Considering the formula as a function of Z for A constant, and neglecting δ, find a formula for $Z(A)$ for the case when the mass is a minimum (ignoring the fact that Z, A are actually integers). Show that for small A this predicts $Z \simeq A/2$. What do you predict for large A?

   (c) * Using the formula for $Z(A)$ found above make a plot [using mathematica or an equivalent program] of $B(Z, A)/A$ for A up to 240. Comment on any discrepancies between the plot you have done and the plots given in the lecture slides.

   (d) * Make a 3d plot of $B(Z, A)/A$ as a function of N, Z and compare it to the table of nuclides. Discuss your findings.
3. For nuclei with $(N, Z) = (50, 28), (82, 50)$ (i.e., 'doubly magic' nuclei) calculate the binding energies in two ways: a) from the definition of B and by looking up atomic masses for the nuclei, and b) using the SEMF. What do you notice about your answers?
4. Use the SEMF to predict the *nuclear* masses of ^{4}He, ^{14}C,^{196}Au, ^{244}Pu in atomic mass units. Compare these to the real values and estimate the percentage error in each case.
5. In the notes we justified the asymmetry term using evenly spaced energy levels and the Pauli Exclusion Principle. Review this argument, and justify the cumulative effect sequence $1, 2, 5, 8, 13, 18, 25, \ldots$. Then show that the energy change must be $\propto (N - Z)^2$. [You can look up on wikipedia a derivation of this based on modelling the nucleus as a Fermi ball of protons and neutrons.]
6. Roughly sketch the distribution of mass estimated from the SEMF as a function of Z for $A = 125$ and $A = 128$ around their minima $Z \simeq 52$ and 54 respectively.
7. Consider isotopes of $_{56}$Ba, with $72 \leq N \leq 88$. Look up data for the neutron separation energy and plot this on a graph as a function of N. Describe the features you see with reference to the SEMF. What feature can't the SEMF explain?

Some (potentially) useful information: The radius of a nuclei may be approximated by $R \approx 1.2A^{1/3}$ fm. The semi-empirical mass formula (SEMF) for the binding energy of a nucleon is

$$B(Z, A) = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(Z, A)\,.$$

Constants in the SEMF: $a_V = 15.56, a_S = 17.23, a_C = 0.697, a_A = 23.28, a_P = 12.0$ where each number is in MeV.

Nuclear Shells: Protons

$$1s_{\frac{1}{2}} \Big\downarrow_{2} 1p_{\frac{3}{2}} \mid 1p_{\frac{1}{2}} \Big\downarrow_{8} 1d_{\frac{5}{2}} \mid 2s_{\frac{1}{2}} \mid 1d_{\frac{3}{2}} \Big\downarrow_{20} 1f_{\frac{7}{2}} \Big\downarrow_{28} 2p_{\frac{3}{2}} \mid 1f_{\frac{5}{2}} \mid 2p_{\frac{1}{2}} \mid 1g_{\frac{9}{2}} \Big\downarrow_{50} 1g_{\frac{7}{2}} \mid 2d_{\frac{5}{2}} \mid 1h_{\frac{11}{2}} \mid 2d_{\frac{3}{2}} \mid 3s_{\frac{1}{2}} \Big\downarrow_{82} 1h_{\frac{9}{2}} \mid 2f_{\frac{7}{2}} \mid \cdots$$

Shells: Neutrons

$$1s_{\frac{1}{2}} \Big\downarrow_{2} 1p_{\frac{3}{2}} \mid 1p_{\frac{1}{2}} \Big\downarrow_{8} 1d_{\frac{5}{2}} \mid 2s_{\frac{1}{2}} \mid 1d_{\frac{3}{2}} \Big\downarrow_{20} 1f_{\frac{7}{2}} \Big\downarrow_{28} 2p_{\frac{3}{2}} \mid 1f_{\frac{5}{2}} \mid 2p_{\frac{1}{2}} \mid 1g_{\frac{9}{2}} \Big\downarrow_{50} 2d_{\frac{5}{2}} \mid 1g_{\frac{7}{2}} \mid 1h_{\frac{11}{2}} \mid 2d_{\frac{3}{2}} \mid 3s_{\frac{1}{2}} \Big\downarrow_{82} 2f_{\frac{7}{2}} \mid 1h_{\frac{9}{2}} \mid \cdots$$

$\dfrac{e^2}{4\pi\varepsilon_0}$	$= 1.439965\,\mathrm{MeV\,fm}$
Boltzmann's constant	$k_B = 8.6173303 \times 10^{-5}\,\mathrm{eV/K}$
Planck's constant	$h = 4.135668 \times 10^{-15}\,\mathrm{eV\,s}$
Speed of light	$c = 2.99792 \times 10^{8}\,\mathrm{m/s}$
Neutrino mean lifetime	881 s
Atomic mass unit	$1\,u = 931.4940954\,\mathrm{MeV}/c^2 = 1.66054 \times 10^{-27}\,\mathrm{kg}$
Mass of electron	$m_e = 5.4858 \times 10^{-4}\,u = 0.51099895\,\mathrm{MeV}/c^2$
Mass of proton	$m_p = 1.00727646688\,u = 938.27208\,\mathrm{MeV}/c^2$
Mass of neutron	$m_n = 1.00866491578\,u = 939.56541\,\mathrm{MeV}/c^2$
Mass of $^{1}_{1}\mathrm{H}$	$= 1.00782503\,u$
Mass of $^{2}_{1}\mathrm{H}$	$= 2.01410178\,u$
Mass of $^{3}_{1}\mathrm{H}$	$= 3.01604927\,u$
Mass of $^{3}_{2}\mathrm{He}$	$= 3.01602932\,u$
Mass of $^{4}_{2}\mathrm{He}$	$= 4.00260325\,u$
Mass of $^{232}_{90}\mathrm{Th}$	$= 232.038055\,u$
Mass of $^{234}_{90}\mathrm{Th}$	$= 234.043601\,u$
Mass of $^{235}_{92}\mathrm{U}$	$= 235.043930\,u$
Mass of $^{236}_{92}\mathrm{U}$	$= 236.045568\,u$
Mass of $^{238}_{92}\mathrm{U}$	$= 238.050788\,u$
Mass of $^{239}_{92}\mathrm{U}$	$= 239.054293\,u$
Mass of $^{240}_{94}\mathrm{Pu}$	$= 240.053811\,u$
Mass of $^{241}_{94}\mathrm{Pu}$	$= 241.056849\,u$
Mass of $^{242}_{94}\mathrm{Pu}$	$= 242.058741\,u$
Mass of the Sun	$M_\odot = 1.988 \times 10^{30}\,\mathrm{kg}$
Gravitational constant	$G = 6.67408 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$

Nuclei masses given are atomic masses.

You can look up other nuclear data from websites
`https://www-nds.iaea.org/relnsd/vcharthtml/VChartHTML.html`
`http://www.nndc.bnl.gov/nudat2/`
`http://atom.kaeri.re.kr/nuchart/`
`http://people.physics.anu.edu.au/~ecs103/chart/`