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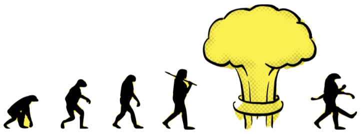


SPA5302

EXERCISE SHEET 1

NUCLEAR PHYSICS & ASTROPHYSICS

2019

Hint: When calculating masses and binding energies, you often need to carry a lot of significant figures to get the correct answer – think about why and where. You also need to be careful that you are using *atomic* or *nuclear* masses in the correct context.

1. (a) The radius of a neutron is approximately 0.8fm. Estimate its density stating any assumptions you make. Imagine a clump of neutrons the size of your head [radius 10 cm] – what would it weigh? What is this mass comparable to?

   (b) A neutron star is a collapsed star made almost entirely of neutrons, more or less pressed together. If a neutron star were to weigh twice the mass of the sun ($1M_\odot \approx 2 \times 10^{30}$ kg), what would its radius be?

Solution:

density = mass/volume$\sim 3m/4\pi r^3$ assuming spherical. $m_n \approx 1.6750 \times 10^{-27}$ kg giving density $\sim 7.81 \times 10^{17}$kg/m^3.

For a head-sized sphere, mass$\approx 7.81 \times 10^{17}kg/m^3 \times 4/3\pi(10\text{cm})^3 \approx 3.3 \times 10^{15}$ kg. which is pretty heavy!

For the neutron star question, by inspection we see it must have a radius $\sim 10^5$ times the head example (why?) giving a radius ~ 10 km.

2. Make a graph of a table of nuclides with the neutron number along the x-axis and proton number along the y-axis [careful - you'll see this plotted in different ways]. On this graph, sketch the line with $Z = N$ up to $N = 20$. Then mark the positions of ^{1}H, ^{2}H, ^{3}He, ^{4}He, ^{12}C,^{14}C, ^{14}N, ^{14}O, ^{18}O. Which of these are isotopes and which are isotones?

Solution:

In order given the nuclei have coordinates $(N, Z) = (0, 1), (1, 1),$ $(1, 2), (2, 2)$, $(6, 6), (8, 6), (7, 7), (6, 8), (10, 8)$. Isotopes have the same Z and chemical symbol. Isotones have the same N: ^{2}H and ^{3}He; and ^{12}C and ^{14}O.

3. The most abundant form of Lithium is ^{7}Li. Write down 4 neighbouring isotopes and isotones of Lithium in the same notation, stating how many neutrons and protons each contains.

Solution: isotopes of $^7_3$Li are $^4_3$Li, $^5_3$Li, $^6_3$Li, $^8_3$Li. All have 3 protons, and $A - 3$ neutrons

isotones are ^{5}H, ^{6}He, ^{8}Be, ^{9}B. All have 4 neutrons, and $A - 4$ protons.

4. In the experiment of Rutherford, Geiger and Marsden an alpha particle (^{4}He nucleus) of energy 7.7 MeV was incident on a gold nucleus ^{197}Au. Estimate the distance of closest approach for a head-on collision. [Hint: Use conservation of energy.]

   Later experiments used the same energy of alpha particles on ^{27}Al and they noted that the simple Coulomb scattering picture broke down for backscattered alpha particles. Use this information to estimate the size of the aluminium nucleus. What approximations have you made?

Solution:

Conservation of energy implies that the initial kinetic energy becomes Coulomb potential energy at the closest approach: $qQ/4\pi\epsilon_0 r = 7.7\,\text{MeV}$. With $qQ = 79 \times 2e^2$ and $e^2/4\pi\epsilon_0 \approx 1.44\text{MeV fm}$ gives $r \approx 79 \times 2 \times 1.44/7.7\text{fm} {=}{\approx}\ 29.5\text{fm}$.

For Al just replace 79 with 13 above to give $r \approx 4.86\text{fm}$. Since something went wrong in the Coulomb approximation, this implies that the nuclear force is important at this radius. We have made approximations of spherical symmetry and neglecting the nuclear force.

5. The total mass of the Earth plus you while you're on the Earth is lighter than the sum of the masses of the Earth and you. Why and by how much? [Hint: assume the gravitational binding energy is GMm/r.]

Solution: This is the binding energy converted to a mass, which is $GMm/c^2r \sim 50\mu\text{g}$ for a person.

6. Make a graph of a table of nuclides with the neutron number along the horizontal axis and proton number along the vertical axis
   (a) On this graph, sketch the line with $Z = N$ up to $N = 150$. On this plot the position of ^{238}U which is the heaviest naturally occurring nuclide. Plot also ^{207}Pb, ^{107}Ag, ^{27}Al and ^{12}C. State how many neutrons are in each.
   (b) The nuclear magic numbers are 2, 8, 20, 28, 50, 82, and 126: A nuclide where the number of protons or neutrons takes one of these values has particularly strong binding energy. On your diagram sketch the possible locations of such nuclei. Identify and name the nucleons in which both the number of protons *and* neutrons are magic.

Solution: a) $^{238}\text{U}={}^{238}_{92}\text{U}_{146}$, $^{207}\text{Pb}={}^{207}_{82}\text{Pb}_{125}$, $^{107}\text{Ag}={}^{107}_{47}\text{Ag}_{60}$, $^{27}\text{Al}={}^{27}_{13}\text{Al}_{14}$, $^{12}\text{C}={}^{12}_{6}\text{C}_{6}$. Coordinates are (N, Z).

b) The magic numbers correspond to vertical or horizontal lines with N or Z taking on values 2, 8, 20, 28, 50, 82, and 126. These cross at 'doubly magic nuclei'. Examples of double magic isotopes include helium-4, oxygen-16, calcium-40, calcium-48, nickel-48, nickel-78, and lead-208.

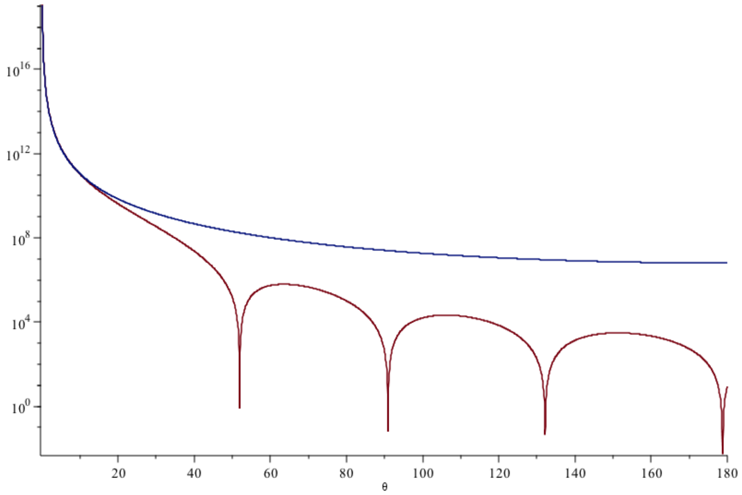
7. Calculate the form factor for a hard sphere given in lectures. Using Maple, Mathematica or Matlab (or something similar) plot the overall differential cross section for this form factor with a log scale on the y-axis, using $2mvR/\hbar = 20$. Compare this to some measured cross sections and comment on your answer.

Solution:

The trick is to use integration by parts:

$$F(q) = \frac{4\pi}{q}\int_0^\infty dr\,\rho(r) r \sin qr = \frac{4\pi\rho}{q}\int_0^R dr\, r \sin qr = \frac{4\pi\rho}{q}\frac{\sin(qR) - Rq\cos(qR)}{q^2} \tag{1}$$

where $q \simeq 2mv/\hbar \sin\theta/2$.



The brown curve shows the form factor squared times the Rutherford formula for $2mvR/\hbar = 20$. The absolute scale of the y-axis is arbitrary. The black curve is the Rutherford formula. Measured cross-sections don't go to zero reflecting blurred surface of nucleus.

8. (a) Calculate the binding energy of $^{238}_{92}$U, given that its *atomic* mass is 238.051 u.
   (b) $^{238}_{92}$U emits an alpha particle and becomes $^{234}_{90}$Th (atomic mass 234.044 u). How much energy is released in this process?

Solution:

a) $B(92, 238) = [92m_H + (238 - 92)m_n - 238.051] \times 931.5$ MeV = 1801.5156 MeV using $m_H = 1.007825, m_n = 1.008665$. Note to use m_H for atomic mass not m_p.

b) The mass of ^{4}He=4.0026 u so the energy released is mass of $U - Th - \alpha$ converted to energy: $(238.051 - 234.044 - 4.0026) \times 931.5$MeV= 4.098MeV. Note that all masses should be either atomic or nuclear – mixing them up gives the wrong value.

9. Use the SEMF to predict the *atomic* masses of ^{14}O,^{107}Ag, in atomic mass units. Compare these to the real values and estimate the percentage error in each case.

Solution:

^{14}O: predicted: 14.00925965u, real: 14.008596u, error 0.0047%

^{107}Ag: predicted: 106.8974466u, real: 106.905091 u, error 0.0071%

Very accurate!

Some (potentially) useful information: The radius of a nuclei may be approximated by $R \approx 1.2A^{1/3}$ fm.
The semi-empirical mass formula (SEMF) for the binding energy of a nucleon is

$$B(Z,A) = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(Z,A)\,.$$

Constants in the SEMF: $a_V = 15.56, a_S = 17.23, a_C = 0.697, a_A = 23.28, a_P = 12.0$ where each number is in MeV.

Nuclear Shells: Protons

$$1s_{\frac{1}{2}}\Big\downarrow_{2} 1p_{\frac{3}{2}} \,|\, 1p_{\frac{1}{2}}\Big\downarrow_{8} 1d_{\frac{5}{2}} \,|\, 2s_{\frac{1}{2}} \,|\, 1d_{\frac{3}{2}}\Big\downarrow_{20} 1f_{\frac{7}{2}}\Big\downarrow_{28} 2p_{\frac{3}{2}} \,|\, 1f_{\frac{5}{2}} \,|\, 2p_{\frac{1}{2}} \,|\, 1g_{\frac{9}{2}}\Big\downarrow_{50} 1g_{\frac{7}{2}} \,|\, 2d_{\frac{5}{2}} \,|\, 1h_{\frac{11}{2}} \,|\, 2d_{\frac{3}{2}} \,|\, 3s_{\frac{1}{2}}\Big\downarrow_{82} 1h_{\frac{9}{2}} \,|\, 2f_{\frac{7}{2}} \,|\, \cdots$$

Shells: Neutrons

$$1s_{\frac{1}{2}}\Big\downarrow_{2} 1p_{\frac{3}{2}} \,|\, 1p_{\frac{1}{2}}\Big\downarrow_{8} 1d_{\frac{5}{2}} \,|\, 2s_{\frac{1}{2}} \,|\, 1d_{\frac{3}{2}}\Big\downarrow_{20} 1f_{\frac{7}{2}}\Big\downarrow_{28} 2p_{\frac{3}{2}} \,|\, 1f_{\frac{5}{2}} \,|\, 2p_{\frac{1}{2}} \,|\, 1g_{\frac{9}{2}}\Big\downarrow_{50} 2d_{\frac{5}{2}} \,|\, 1g_{\frac{7}{2}} \,|\, 1h_{\frac{11}{2}} \,|\, 2d_{\frac{3}{2}} \,|\, 3s_{\frac{1}{2}}\Big\downarrow_{82} 2f_{\frac{7}{2}} \,|\, 1h_{\frac{9}{2}} \,|\, \cdots$$

$\frac{e^2}{4\pi\varepsilon_0}$	$= 1.439965\,\mathrm{MeV\,fm}$
Boltzmann's constant	$k_B = 8.6173303 \times 10^{-5}\,\mathrm{eV/K}$
Planck's constant	$h = 4.135668 \times 10^{-15}\,\mathrm{eV\,s}$
Speed of light	$c = 2.99792 \times 10^{8}\,\mathrm{m/s}$
Neutrino mean lifetime	881 s
Atomic mass unit	$1\,u = 931.4940954\,\mathrm{MeV}/c^2 = 1.66054 \times 10^{-27}\,\mathrm{kg}$
Mass of electron	$m_e = 5.4858 \times 10^{-4}\,u = 0.51099895\,\mathrm{MeV}/c^2$
Mass of proton	$m_p = 1.00727646688\,u = 938.27208\,\mathrm{MeV}/c^2$
Mass of neutron	$m_n = 1.00866491578\,u = 939.56541\,\mathrm{MeV}/c^2$
Mass of $^{1}_{1}$H	$= 1.00782503\,u$
Mass of $^{2}_{1}$H	$= 2.01410178\,u$
Mass of $^{3}_{1}$H	$= 3.01604927\,u$
Mass of $^{3}_{2}$He	$= 3.01602932\,u$
Mass of $^{4}_{2}$He	$= 4.00260325\,u$
Mass of $^{232}_{90}$Th	$= 232.038055\,u$
Mass of $^{234}_{90}$Th	$= 234.043601\,u$
Mass of $^{235}_{92}$U	$= 235.043930\,u$
Mass of $^{236}_{92}$U	$= 236.045568\,u$
Mass of $^{238}_{92}$U	$= 238.050788\,u$
Mass of $^{239}_{92}$U	$= 239.054293\,u$
Mass of $^{240}_{94}$Pu	$= 240.053811\,u$
Mass of $^{241}_{94}$Pu	$= 241.056849\,u$
Mass of $^{242}_{94}$Pu	$= 242.058741\,u$
Mass of the Sun	$M_\odot = 1.988 \times 10^{30}\,\mathrm{kg}$
Gravitational constant	$G = 6.67408 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$

Nuclei masses given are atomic masses.

You can look up other nuclear data from websites
`https://www-nds.iaea.org/relnsd/vcharthtml/VChartHTML.html`
`http://www.nndc.bnl.gov/nudat2/`
`http://atom.kaeri.re.kr/nuchart/`
`http://people.physics.anu.edu.au/~ecs103/chart/`