Queen Mary University of London

SPA5302 *NUCLEAR PHYSICS & ASTROPHYSICS 2019*

HOMEWORK SET 2 HAND IN: FRIDAY 25 OCTOBER BY 4PM

Please write your answers clearly and neatly. **Include your working!** *Answers without proper reasoning will receive no marks. For some questions you will need to look up atomic masses, periodic tables etc.*

Hand in solutions to ALL questions. ONE of these will be marked and will count towards your class record.

1. In the SEMF we wrote the Coulomb term as $a_C Z^2/A^{1/3}$. Derive an expression for a_C based on the electrostatic formula for the potential energy of a charged sphere of charge Q and radius R: [10]

$$E = \frac{3}{5}\frac{Q^2}{4\pi\varepsilon_0 R}\,.$$

   Use this formula to estimate the theoretical value of a_C and compare it to its measured value.

2. Sketch energy level diagrams using the shell model for the mirror nuclei ${}^{17}_{8}\mathrm{O}$ and ${}^{17}_{9}\mathrm{F}$, clearly labelling each energy level. Identify the magic levels in each case. Give the ground state spin and parity on both cases. [10]

3. Give the nuclear configuration of the following nuclei [10]

$$^{34}\mathrm{S}, {}^{35}\mathrm{S}, {}^{109}\mathrm{In}, {}^{111}\mathrm{Sb},.$$

   Hence give the nuclear state J^π of each. What can you say about the state of ${}^{38}\mathrm{Cl}$?

4. In the spin orbit coupling term the dot product $\boldsymbol{L}\cdot\boldsymbol{S}$ appears. Derive a formula for its expectation value, and hence calculate the difference in energy levels induced by this term. [10]

Some (potentially) useful information: The radius of a nuclei may be approximated by $R \approx 1.2A^{1/3}$ fm. The semi-empirical mass formula (SEMF) for the binding energy of a nucleon is

$$B(Z,A) = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(Z,A)\,.$$

Constants in the SEMF: $a_V = 15.56, a_S = 17.23, a_C = 0.697, a_A = 23.28, a_P = 12.0$ where each number is in MeV.

Nuclear Shells: Protons

$$1s_{\frac{1}{2}}\Big|_2 1p_{\frac{3}{2}} \mid 1p_{\frac{1}{2}}\Big|_8 1d_{\frac{5}{2}} \mid 2s_{\frac{1}{2}} \mid 1d_{\frac{3}{2}}\Big|_{20} 1f_{\frac{7}{2}}\Big|_{28} 2p_{\frac{3}{2}} \mid 1f_{\frac{5}{2}} \mid 2p_{\frac{1}{2}} \mid 1g_{\frac{9}{2}}\Big|_{50} 1g_{\frac{7}{2}} \mid 2d_{\frac{5}{2}} \mid 1h_{\frac{11}{2}} \mid 2d_{\frac{3}{2}} \mid 3s_{\frac{1}{2}}\Big|_{82} 1h_{\frac{9}{2}} \mid 2f_{\frac{7}{2}} \mid \cdots$$

Shells: Neutrons

$$1s_{\frac{1}{2}}\Big|_2 1p_{\frac{3}{2}} \mid 1p_{\frac{1}{2}}\Big|_8 1d_{\frac{5}{2}} \mid 2s_{\frac{1}{2}} \mid 1d_{\frac{3}{2}}\Big|_{20} 1f_{\frac{7}{2}}\Big|_{28} 2p_{\frac{3}{2}} \mid 1f_{\frac{5}{2}} \mid 2p_{\frac{1}{2}} \mid 1g_{\frac{9}{2}}\Big|_{50} 2d_{\frac{5}{2}} \mid 1g_{\frac{7}{2}} \mid 1h_{\frac{11}{2}} \mid 2d_{\frac{3}{2}} \mid 3s_{\frac{1}{2}}\Big|_{82} 2f_{\frac{7}{2}} \mid 1h_{\frac{9}{2}} \mid \cdots$$

$\frac{e^2}{4\pi\varepsilon_0}$	$= 1.439965\,\text{MeV fm}$
Boltzmann's constant	$k_B = 8.6173303 \times 10^{-5}\,\text{eV/K}$
Planck's constant	$h = 4.135668 \times 10^{-15}\,\text{eV s}$
Speed of light	$c = 2.99792 \times 10^{8}\,\text{m/s}$
Neutrino mean lifetime	$881\,\text{s}$
Atomic mass unit	$1\,u = 931.4940954\,\text{MeV}/c^2 = 1.66054 \times 10^{-27}\,\text{kg}$
Mass of electron	$m_e = 5.4858 \times 10^{-4}\,u = 0.51099895\,\text{MeV}/c^2$
Mass of proton	$m_p = 1.00727646688\,u = 938.27208\,\text{MeV}/c^2$
Mass of neutron	$m_n = 1.00866491578\,u = 939.56541\,\text{MeV}/c^2$
Mass of $^{1}_{1}$H	$= 1.00782503\,u$
Mass of $^{2}_{1}$H	$= 2.01410178\,u$
Mass of $^{3}_{1}$H	$= 3.01604927\,u$
Mass of $^{3}_{2}$He	$= 3.01602932\,u$
Mass of $^{4}_{2}$He	$= 4.00260325\,u$
Mass of $^{232}_{90}$Th	$= 232.038055\,u$
Mass of $^{234}_{90}$Th	$= 234.043601\,u$
Mass of $^{235}_{92}$U	$= 235.043930\,u$
Mass of $^{236}_{92}$U	$= 236.045568\,u$
Mass of $^{238}_{92}$U	$= 238.050788\,u$
Mass of $^{239}_{92}$U	$= 239.054293\,u$
Mass of $^{240}_{94}$Pu	$= 240.053811\,u$
Mass of $^{241}_{94}$Pu	$= 241.056849\,u$
Mass of $^{242}_{94}$Pu	$= 242.058741\,u$
Mass of the Sun	$M_\odot = 1.988 \times 10^{30}\,\text{kg}$
Gravitational constant	$G = 6.67408 \times 10^{-11}\,\text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2}$

Nuclei masses given are atomic masses.

You can look up other nuclear data from websites
`https://www-nds.iaea.org/relnsd/vcharthtml/VChartHTML.html`
`http://www.nndc.bnl.gov/nudat2/`
`http://atom.kaeri.re.kr/nuchart/`
`http://people.physics.anu.edu.au/~ecs103/chart/`