



Hint: When calculating masses and binding energies, you often need to carry a lot of significant figures to get the correct answer – think about why and where. You also need to be careful that you are using *atomic* or *nuclear* masses in the correct context.

1. (a) The radius of a neutron is approximately 0.8fm. Estimate its density stating any assumptions you make. Imagine a clump of neutrons the size of your head [radius 10 cm] – what would it weigh? What is this mass comparable to?
- (b) A neutron star is a collapsed star made almost entirely of neutrons, more or less pressed together. If a neutron star were to weigh twice the mass of the sun ($1M_{\odot} \approx 2 \times 10^{30}$ kg), what would its radius be?

2. Make a graph of a table of nuclides with the neutron number along the x -axis and proton number along the y -axis [careful - you'll see this plotted in different ways]. On this graph, sketch the line with $Z = N$ up to $N = 20$. Then mark the positions of ${}^1\text{H}$, ${}^2\text{H}$, ${}^3\text{He}$, ${}^4\text{He}$, ${}^{12}\text{C}$, ${}^{14}\text{C}$, ${}^{14}\text{N}$, ${}^{16}\text{O}$, ${}^{18}\text{O}$. Which of these are isotopes and which are isotones?

3. The most abundant form of Lithium is ${}^7\text{Li}$. Write down 4 neighbouring isotopes and isotones of Lithium in the same notation, stating how many neutrons and protons each contains.

4. In the experiment of Rutherford, Geiger and Marsden an alpha particle (${}^4\text{He}$ nucleus) of energy 7.7 MeV was incident on a gold nucleus ${}^{197}\text{Au}$. Estimate the distance of closest approach for a head-on collision. [Hint: Use conservation of energy.]

Later experiments used the same energy of alpha particles on ${}^{27}\text{Al}$ and they noted that the simple Coulomb scattering picture broke down for backscattered alpha particles. Use this information to estimate the size of the aluminium nucleus. What approximations have you made?

5. The total mass of the Earth plus you while you're on the Earth is lighter than the sum of the masses of the Earth and you. Why and by how much? [Hint: assume the gravitational binding energy is GMm/r .]

6. Make a graph of a table of nuclides with the neutron number along the horizontal axis and proton number along the vertical axis

- (a) On this graph, sketch the line with $Z = N$ up to $N = 150$. On this plot the position of ${}^{238}\text{U}$ which is the heaviest naturally occurring nuclide. Plot also ${}^{207}\text{Pb}$, ${}^{107}\text{Ag}$, ${}^{27}\text{Al}$ and ${}^{12}\text{C}$. State how many neutrons are in each.

- (b) The nuclear magic numbers are 2, 8, 20, 28, 50, 82, and 126: A nuclide where the number of protons or neutrons takes one of these values has particularly strong binding energy. On your diagram sketch the possible locations of such nuclei. Identify and name the nucleons in which both the number of protons *and* neutrons are magic.

7. Calculate the form factor for a hard sphere given in lectures. Using Maple, Mathematica or Matlab (or something similar) plot the overall differential cross section for this form factor with a log scale on the y -axis, using $2mvR/\hbar = 20$. Compare this to some measured cross sections and comment on your answer.
8. (a) Calculate the binding energy of ${}^{238}_{92}\text{U}$, given that its *atomic* mass is 238.051 u.
 (b) ${}^{238}_{92}\text{U}$ emits an alpha particle and becomes ${}^{234}_{90}\text{Th}$ (atomic mass 234.044 u). How much energy is released in this process?
9. Use the SEMF to predict the *atomic* masses of ${}^{14}\text{O}$, ${}^{107}\text{Ag}$, in atomic mass units. Compare these to the real values and estimate the percentage error in each case.

Some (potentially) useful information: The radius of a nuclei may be approximated by $R \approx 1.2A^{1/3}$ fm. The semi-empirical mass formula (SEMF) for the binding energy of a nucleon is

$$B(Z, A) = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(Z, A).$$

Constants in the SEMF: $a_V = 15.56$, $a_S = 17.23$, $a_C = 0.697$, $a_A = 23.28$, $a_P = 12.0$ where each number is in MeV.

Nuclear Shells: Protons

$$1s_{\frac{1}{2}} \downarrow_2 \quad 1p_{\frac{3}{2}} \downarrow_4 \quad 1p_{\frac{1}{2}} \downarrow_2 \quad 1d_{\frac{5}{2}} \downarrow_8 \quad 2s_{\frac{1}{2}} \downarrow_2 \quad 1d_{\frac{3}{2}} \downarrow_6 \quad 1f_{\frac{7}{2}} \downarrow_{14} \quad 2p_{\frac{3}{2}} \downarrow_6 \quad 1f_{\frac{5}{2}} \downarrow_8 \quad 2p_{\frac{1}{2}} \downarrow_2 \quad 1g_{\frac{9}{2}} \downarrow_{18} \quad 1g_{\frac{7}{2}} \downarrow_{10} \quad 2d_{\frac{5}{2}} \downarrow_{10} \quad 1h_{\frac{11}{2}} \downarrow_{12} \quad 2d_{\frac{3}{2}} \downarrow_6 \quad 3s_{\frac{1}{2}} \downarrow_2 \quad 1h_{\frac{9}{2}} \downarrow_{10} \quad 2f_{\frac{7}{2}} \downarrow_{14} \quad \dots$$

Shells: Neutrons

$$1s_{\frac{1}{2}} \downarrow_2 \quad 1p_{\frac{3}{2}} \downarrow_4 \quad 1p_{\frac{1}{2}} \downarrow_2 \quad 1d_{\frac{5}{2}} \downarrow_8 \quad 2s_{\frac{1}{2}} \downarrow_2 \quad 1d_{\frac{3}{2}} \downarrow_6 \quad 1f_{\frac{7}{2}} \downarrow_{14} \quad 2p_{\frac{3}{2}} \downarrow_6 \quad 1f_{\frac{5}{2}} \downarrow_8 \quad 2p_{\frac{1}{2}} \downarrow_2 \quad 1g_{\frac{9}{2}} \downarrow_{18} \quad 2d_{\frac{5}{2}} \downarrow_{10} \quad 1g_{\frac{7}{2}} \downarrow_{10} \quad 1h_{\frac{11}{2}} \downarrow_{12} \quad 2d_{\frac{3}{2}} \downarrow_6 \quad 3s_{\frac{1}{2}} \downarrow_2 \quad 2f_{\frac{7}{2}} \downarrow_{14} \quad 1h_{\frac{9}{2}} \downarrow_{10} \quad \dots$$

$\frac{e^2}{4\pi\epsilon_0}$	= 1.439965 MeV fm
Boltzmann's constant	$k_B = 8.6173303 \times 10^{-5}$ eV/K
Planck's constant	$h = 4.135668 \times 10^{-15}$ eV s
Speed of light	$c = 2.99792 \times 10^8$ m/s
Neutrino mean lifetime	881 s
Atomic mass unit	$1 u = 931.4940954 \text{ MeV}/c^2 = 1.66054 \times 10^{-27}$ kg
Mass of electron	$m_e = 5.4858 \times 10^{-4} u = 0.51099895 \text{ MeV}/c^2$
Mass of proton	$m_p = 1.00727646688 u = 938.27208 \text{ MeV}/c^2$
Mass of neutron	$m_n = 1.00866491578 u = 939.56541 \text{ MeV}/c^2$
Mass of ${}^1_1\text{H}$	= 1.00782503 u
Mass of ${}^2_1\text{H}$	= 2.01410178 u
Mass of ${}^3_1\text{H}$	= 3.01604927 u
Mass of ${}^3_2\text{He}$	= 3.01602932 u
Mass of ${}^4_2\text{He}$	= 4.00260325 u
Mass of ${}^{232}_{90}\text{Th}$	= 232.038055 u
Mass of ${}^{234}_{90}\text{Th}$	= 234.043601 u
Mass of ${}^{235}_{92}\text{U}$	= 235.043930 u
Mass of ${}^{236}_{92}\text{U}$	= 236.045568 u
Mass of ${}^{238}_{92}\text{U}$	= 238.050788 u
Mass of ${}^{239}_{92}\text{U}$	= 239.054293 u
Mass of ${}^{240}_{94}\text{Pu}$	= 240.053811 u
Mass of ${}^{241}_{94}\text{Pu}$	= 241.056849 u
Mass of ${}^{242}_{94}\text{Pu}$	= 242.058741 u
Mass of the Sun	$M_{\odot} = 1.988 \times 10^{30}$ kg
Gravitational constant	$G = 6.67408 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$

Nuclei masses given are atomic masses.

You can look up other nuclear data from websites
<https://www-nds.iaea.org/relnsd/vcharthtml/VChartHTML.html>
<http://www.nndc.bnl.gov/nudat2/>
<http://atom.kaeri.re.kr/nuchart/>
<http://people.physics.anu.edu.au/~ecs103/chart/>