

# Physical Cosmology Exercises

Phil Bull, September 23, 2019

Week	Title	Release date	Hand-in date	Marks returned
1	Expanding universe	Tue 24 Sep	<i>Not assessed</i>	<i>Not assessed</i>
2	Geometry and distance	Tue 01 Oct	Wed 09 Oct	Tue 15 Oct
3	Friedmann equation	Tue 08 Oct	Wed 16 Oct	Tue 22 Oct
4	Acceleration and the cosmic horizon	Tue 15 Oct	Wed 23 Oct	Tue 29 Oct
5	Thermal history of the Universe	Tue 22 Oct	<i>Not assessed</i>	<i>Not assessed</i>
6	Cosmic Microwave Background	Tue 29 Oct	Wed 13 Nov	Tue 19 Nov
7	READING WEEK	n/a	n/a	n/a
8	Inflation	Tue 12 Nov	Wed 20 Nov	Tue 26 Nov
9	Dark matter	Tue 19 Nov	Wed 27 Nov	Tue 03 Dec
10	Structure formation	Tue 26 Nov	Wed 04 Dec	Tue 10 Dec
11	Observational cosmology	Tue 03 Dec	<i>Not assessed</i>	<i>Not assessed</i>
12	REVISION	Tue 10 Dec	<i>Not assessed</i>	<i>Not assessed</i>

## Exam question pro-tips\*

- Show your working;
- Make it clear what your final answer is (e.g. underline it);
- Write neatly and legibly;
- Diagrams/sketches should be neat and properly labelled, but don't need to be super precise (no need for a ruler);
- Don't forget units;
- Use maths symbols (e.g. equals sign, integrals) properly;
- Cross-out wrong answers simply but clearly;
- If a question asks you to explain or discuss something, write a few concise sentences (not just a handful of words, and not a long essay!);
- **Read the instructions fully and carefully before you start writing!**

(\*Based on some common reasons for losing marks in last year's exam.)

## Week 1: Expanding universe

This sheet doesn't need to be handed in for marking, but please bring it (with completed answers!) to next week's tutorial so we can go over the answers.

### 1. Maths practice: Unit conversions

- What is the frequency, in GHz, of electromagnetic radiation with a wavelength of 21cm?
- A typical cluster of galaxies has a mass of around  $10^{14} M_{\odot}$ . What is this in kg?
- What is 6 arcsec. in degrees? How many radians are in a degree?
- Consider a neutrino of energy 0.06 eV in its rest frame. What is its mass in kg?

### 2. History of the Universe

- Give a one-sentence description of what happened during each of the following epochs:  
(i) Inflation; (ii) Structure formation; (iii) Cosmic Dawn.
- Put the following events in chronological order (from earliest to most recent):  
*First neutral atoms formed — Extinction of the dinosaurs — Universe became transparent to light — Stoke City won the League Cup — Inflation — Formation of first stars and galaxies — First nuclei formed — Large-scale structure formed.*

### 3. Spectroscopy

An emission line is detected at a wavelength of 406.01 nm in a galaxy with a recession velocity of 13,191 km/s.

- What is the redshift of the galaxy?
- A second galaxy of the same type is found at a redshift  $z = 0.34$ . What wavelength will this emission line be observed at for this galaxy?

### 4. Olbers' paradox

Imagine that a static, infinitely-large, infinitely old Universe is uniformly filled with stars, with a density  $n$ . Assume that each star has the same luminosity,  $L$ , and stellar radius,  $R$ .

- Calculate the total solid angle subtended by all the stars in a thin spherical shell of width  $dr$  at a distance  $r$  from Earth.
- Write an expression for the total flux received from each shell.
- Assume that the stars are opaque, so no light can pass through them. Under this assumption, what is the brightness of the night sky?

### Practice exam question: Expansion and redshift

- What redshift corresponds to when the Universe was (i) half its current size; (ii) a tenth its current size?  
[5 marks]
- The Lyman- $\alpha$  ( $n = 2 \rightarrow 1$ ) emission line of hydrogen is emitted at a wavelength of 121.6 nm in the rest frame. Galaxies A and B are observed with Lyman- $\alpha$  emission lines at frequencies of  $0.99 \times 10^{15}$  Hz and  $1.90 \times 10^{15}$  Hz respectively. (i) What is the redshift of each galaxy? (ii) Which galaxy is further from us?  
[10 marks]
- By drawing a diagram, explain Olbers' Paradox. How does the observation that the Universe is expanding solve the paradox? Name one other possible solution and explain why it does not work.  
[10 marks]
- Explain two pieces of evidence for the hot Big Bang model. You should discuss (i) how the observations were made; (ii) how they imply that there was a Big Bang; (iii) how they imply that it was hot; and (iv) whether they can also be used as evidence to support the Steady-State theory or not (explain why).  
[15 marks]