Solution 5.1

(a) The distribution function, f, expresses the density of stars in the 6-dimensional phase space defined by position and velocity.

(b) The collisionless Boltzmann equation states that $\frac{df}{dt} = 0$ i.e. f is constant, locally, along the orbit of a star.
Therefore, as the density of stars in space increases, the density in 'velocity space' must decrease in order to keep f constant. Therefore the local velocity dispersion must increase.

(c) The CBE expresses the change in the distribution function, f, as a function of time, position and velocity, but it is difficult to determine observationally, especially in a distant galaxy, where individual stars cannot be resolved. In contrast, the Jeans equations use parameters such as number density, mean velocities, mean square velocities and velocity dispersions, that are much more easily determined from observations.

(d) Define number density $n = \int f \, d^3\underline{v}$

and $n\langle v_i \rangle = \int v_i f \, d^3\underline{v}$ where $\langle v_i \rangle$ is the mean value of the v_i velocity component

Integrating the CBE over velocity (taking 'zeroth' moment):

$$\int \left[\frac{\partial f}{\partial t} + \sum_{i=1}^{3} \frac{dx_i}{dt}\frac{\partial f}{\partial x_i} + \sum_{i=1}^{3} \frac{dv_i}{dt}\frac{\partial f}{\partial v_i} \right] d^3\underline{v} = 0$$