SPA7010 Example Problems 5
(05 Mar 2020)

Collisionless Boltzmann Equation and Jeans Equations

Problem 5.1

(a) Explain briefly the meaning of the distribution function, f, used in studying the dynamics of stars in galaxies.

(b) A star moves in an elongated orbit about the Galactic centre. As it moves inwards its distance from the Galactic Centre decreases and the star density around it increases. How does f change? How do the velocity dispersions of the stars around it change and why?

(c) What advantage do the Jeans equations have over the collisionless Boltzmann equation in describing the dynamics and densities of stars in observed galaxies?

(d) The collisionless Boltzmann equation gives

$$\frac{\partial f}{\partial t} + \sum_{i=1}^{3} \left(\frac{\mathrm{d}x_i}{\mathrm{d}t} \frac{\partial f}{\partial x_i} + \frac{\mathrm{d}v_i}{\mathrm{d}t} \frac{\partial f}{\partial v_i} \right) = 0 ,$$

where f is the distribution function, t is time, and x_i and v_i (for $i = 1, 2, 3$) are the components of the position vector $\mathbf{x}$ and velocity vector $\mathbf{v}$ respectively.

Derive from this the first of the Jeans equations,

$$\frac{\partial n}{\partial t} + \sum_{i=1}^{3} \frac{\partial}{\partial x_i} (n \langle v_i \rangle) = 0 ,$$

from the collisionless Boltzmann equation, where n is the number density of stars and $\langle v_i \rangle$ is the mean value of the v_i velocity component at a point. Explain your working and assumptions.

(e) The velocity dispersion tensor σ_{ij} is defined by

$$\sigma_{ij}^2 = \frac{1}{n} \int \Big(v_i - \langle v_i \rangle \Big) \Big(v_j - \langle v_j \rangle \Big) \; f \; \mathrm{d}^3\mathbf{v} ,$$

where v_1, v_2 and v_3 are the components of the velocity vector $\mathbf{v}$, n is the number density of stars in space, f is the distribution function, and i and $j = 1, 2$ and 3.

Prove that

$$\sigma_{ij}^2 = \langle v_i v_j \rangle - \langle v_i \rangle \langle v_j \rangle \quad .$$

(f) A spherically-symmetric galaxy consists only of stars (it has no dark matter). It has a density distribution for which the internal potential energy is

$$U = - \frac{G M_{tot}^2}{6a} .$$

If the typical velocity of stars in this galaxy is v, find an expression for the total mass from the virial theorem, in terms of a and v, on the assumption that the system is pressure supported and virialised, and v is constant throughout.