Helpful Tips, Tricks, and other Wonderful things!

- Repeated indices indicate a summation, so you can replace these dummy indices with anything you like:
  \[ \partial_{\mu} \phi \partial_{\nu} \phi = \partial_{\nu} \phi \partial_{\mu} \phi = \partial_{\text{anything}} \phi \partial_{\text{anything}} \phi \]

- \[ X_{\mu} X^{\mu} = \eta_{\mu\nu} X_{\mu} X_{\nu} = X^{\mu} X_{\nu} = X^{\mu} X_{\mu} \]

- \( \partial_{\mu} \) means \( \frac{\partial}{\partial x^{\mu}} \) \( \text{[notice the index placement]} \)

- If you apply a transformation to \( A_{\mu} \), then \( A^{\mu} \) must also be transformed. After all, \( A_{\mu} = \eta^{\mu\nu} A_{\nu} \)

- If you take the derivative of a product of fields, use the product (Leibnitz) rule:
  
  \[
  \begin{align*}
  \frac{\partial}{\partial A_{\nu}} (A_{\mu} A^{\mu}) &= \frac{\partial A_{\mu}}{\partial A_{\nu}} A^{\mu} + A_{\mu} \frac{\partial A^{\mu}}{\partial A_{\nu}} \\
  &= \delta^{\nu}_{\mu} A^{\mu} + A_{\mu} \eta^{\mu\nu} \frac{\partial A_{\nu}}{\partial A_{\nu}} \\
  &= A^{\nu} + A^{\nu} \delta^{\nu}_{\nu} \\
  &= 2 A^{\nu}
  \end{align*}
  \]

- Don't use the same index more than 2 times in a product.
  
  \[ L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^{2}}{2} A_{\mu} A^{\mu} \]

  EOM: \[ \partial_{\mu} \frac{\partial L}{\partial (\partial_{\nu} A_{\mu})} - \frac{\partial L}{\partial A_{\mu}} = 0 \]

  \( \Rightarrow \) I rewrite \( L = -\frac{1}{4} F_{\rho\sigma} F^{\rho\sigma} + \frac{m^{2}}{2} A_{\rho} A^{\rho} \) before taking derivatives.

  i.e. \( \frac{\partial F_{\mu\nu} F^{\mu\nu}}{\partial (\partial_{\nu} A_{\mu})} \) is bad, \( \frac{\partial F_{\rho\sigma} F^{\rho\sigma}}{\partial (\partial_{\mu} A_{\nu})} \) is good
* Keep track of the number and placement of free indices in an equation or expression:

  \[ - \partial_\mu F^{\mu \nu} = m^2 A_\nu \quad \text{bad} \times \]

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  \[ - \partial_\mu F^{\mu \nu} = m^2 A_\nu \quad \text{good} \checkmark \]

  (This is also a useful way to check your intermediate steps and final answer.)

* Be mindful of the objects in an equation or expression: are they vectors, fields, matrices, components of a matrix? Is it meaningful to divide by a field or matrix?

* Here's how repeated indices work for partial derivatives:

  \[ \partial_\mu \phi = (\partial_0 \phi, \partial_1 \phi, \partial_2 \phi, \partial_3 \phi) \]

  \[ = (\partial_0 \phi, \partial_x \phi, \partial_y \phi, \partial_z \phi) = (\partial_0 \phi, \vec{\nabla} \phi) \]

  \[ = (\partial_0 \phi, -\vec{\nabla} \phi) \]

  \[ = (\partial_\mu \phi, \vec{\nabla} \phi) \]

  \[ \Rightarrow \quad \partial_\mu \phi \partial_\mu \phi = \partial_0 \phi \partial_0 \phi + \partial_1 \phi \partial_1 \phi + \partial_2 \phi \partial_2 \phi + \partial_3 \phi \partial_3 \phi \]

  \[ = \partial_0 \phi \partial_0 \phi - \vec{\nabla} \phi \cdot \vec{\nabla} \phi \]

  or equivalently

  \[ = (\partial_0 \phi)^2 - \vec{\nabla} \phi \cdot \vec{\nabla} \phi \]