

MSc/MSci Examination**Main Examination Period 2018****SPA7010P / SPA7010U****The Galaxy****Duration: 2 hours 30 minutes****YOU ARE NOT PERMITTED TO READ THE CONTENTS OF THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR.****Instructions:**

Answer ALL questions from Section A. Answer ONLY TWO questions from Section B. Section A carries 50 marks, each question in section B carries 25 marks.

If you answer more questions than specified, only the first answers (up to the specified number) will be marked. Cross out any answers that you do not wish to be marked.

Only non-programmable calculators are permitted in this examination. Please state on your answer book the name and type of machine used.

Complete all rough workings in the answer book and cross through any work that is not to be assessed.

Important note: The academic regulations state that possession of unauthorised material at any time when a student is under examination conditions is an assessment offence and can lead to expulsion from QMUL.

Please check now to ensure you do not have any notes, mobile phones or unauthorised electronic devices on your person. If you have any, raise your hand and give them to an invigilator immediately. It is also an offence to have any writing of any kind on your person, including on your body. If you are found to have hidden unauthorised material elsewhere, including toilets and cloakrooms it will be treated as being found in your possession. Unauthorised material found on your mobile phone or other electronic device will be considered the same as being in possession of paper notes. A mobile phone that causes a disruption is also an assessment offence.

EXAM PAPERS MUST NOT BE REMOVED FROM THE EXAM ROOM.

Examiners:

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Prof. R. Nelson

SECTION A

Answer ALL questions in Section A

Question A1

Discuss the main observed characteristics of spiral galaxies, referring to bars/bulges, colours, spectra, spiral arms, gas and dust content. Describe how these characteristics vary with Hubble type, from Sa to Sc or SBa to SBc.

[6 marks]**Question A2**

The Tully-Fisher relation for spiral galaxies states that $L \simeq kv_{\text{rot}}^4$, where L is luminosity, v_{rot} is the peak rotational velocity and k is a constant.

One spiral galaxy in the Virgo cluster is observed with $v_{\text{rot}} = 180 \text{ km s}^{-1}$ and apparent magnitude $m_V = 12.0$. Another galaxy, in the Coma cluster, is observed with $v_{\text{rot}} = 240 \text{ km s}^{-1}$ and $m_V = 14.45$. Estimate the ratio of the distances to the two clusters.

[5 marks]**Question A3**

Define the terms “crossing time” and “relaxation time” for a stellar system.

In a large stellar system of N stars, the relaxation time T_{rel} and crossing time T_{cr} are approximately related by

$$\frac{T_{\text{rel}}}{T_{\text{cr}}} \approx \frac{N}{12 \ln N}.$$

For (a) a typical galaxy and (b) a typical globular cluster, quote approximate values for N , the system radius and typical velocity; hence derive T_{cr} and T_{rel} in each case, and state whether two-body encounters are important.

You may use the relation $1 \text{ km s}^{-1} \approx 10^{-6} \text{ pc year}^{-1}$.

[6 marks]**Question A4**

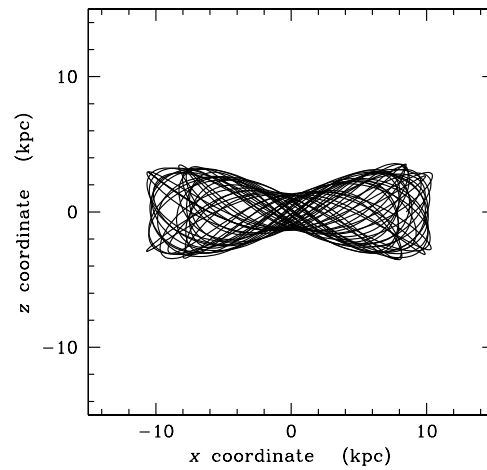
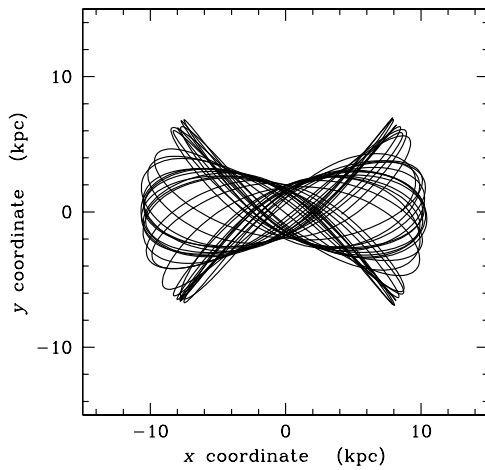
The distribution function $f(\mathbf{x}, \mathbf{v}, t)$ describes the density of stars in the 6-dimensional phase-space of position \mathbf{x} and velocity \mathbf{v} , at time t .

Write down (without proof) expressions for the number density of stars $n(\mathbf{x}, t)$, and the mean velocity component in the i -direction, $\langle v_i \rangle$, in terms of f .

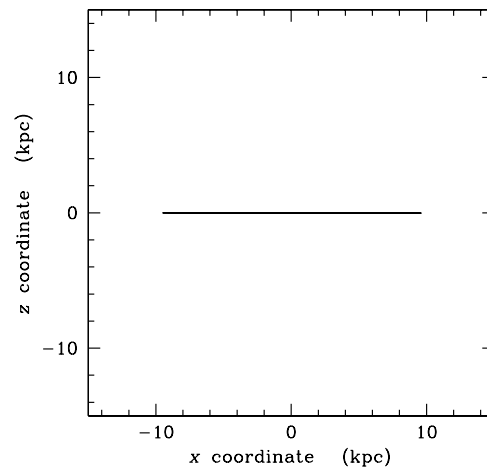
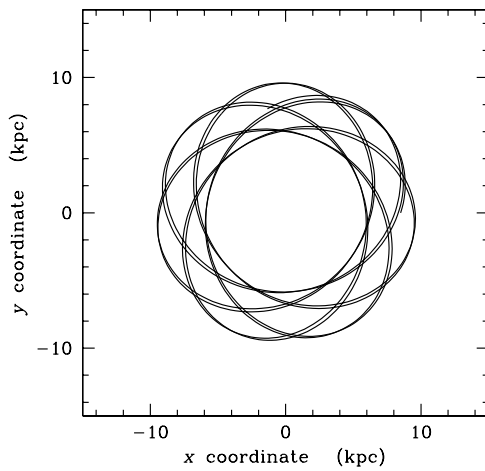
[4 marks]

Question A5

Potential A:



Potential B:



The diagrams above show the orbit of a star in two gravitational potentials, A (upper) and B (lower): each orbit is shown projected in the $x-y$ plane (left) and the $x-z$ plane (right).

What do you conclude about each of the potentials A and B: are they (i) spherical, (ii) flattened (oblate), or (iii) triaxial? Justify your answer on the basis of the characteristics of each orbit.

[4 marks]

Turn over

Question A6

Describe the main properties and physical importance of dust in the interstellar medium: refer to its effect on observations, its composition, how it is observationally detected, and its influence on star formation.

[5 marks]**Question A7**

List any four assumptions of the Simple Model of galactic chemical evolution.

[4 marks]**Question A8**

A star near the Galactic plane has apparent magnitudes in the blue and visual bands of $B = 18.87$, $V = 17.47$. A spectrum of the star indicates that it is a K-dwarf with absolute magnitude $M_V = 7.20$ and unreddened colour index $(B - V)_0 = 1.10$.

Given that the reddening ratio for interstellar dust is $A_V/E(B - V) = 3.0$, estimate (i) the reddening of the star, (ii) the V-band extinction in magnitudes, and (iii) the distance to the star in parsecs.

[4 marks]**Question A9**

Draw a sketch of the light-curve (flux vs time) for a typical gravitational microlensing event.

The Einstein ring radius for gravitational lensing is given by

$$r_E = \sqrt{\frac{4GM}{c^2} \frac{D_{LS} D_L}{D_S}},$$

where M is the lens mass, D_L is the distance from Earth to the lens, D_S is the distance from Earth to the source, D_{LS} is the distance between lens and source, and c is the speed of light.

Assuming a lens mass $M = 0.1 M_\odot$, a source in the Galactic bulge at $D_S = 8 \text{ kpc}$, and the lens positioned half-way between observer and source, calculate the Einstein radius. Express your answer in astronomical units.

[6 marks]**Question A10**

Describe the general features of the components of the Milky Way Galaxy, including the disc, bulge/bar, stellar halo and dark matter halo.

[6 marks]

SECTION B

Answer TWO questions from Section B

Question B1

An isolated system of N stars is bound by the self-gravity of the stars. The i th star has a mass m_i , position vector \mathbf{x}_i and velocity $\dot{\mathbf{x}}_i \equiv d\mathbf{x}_i/dt$ where t is time, and the origin is the centre of mass of the system. The total moment of inertia I of the system is defined as

$$I \equiv \sum_{i=1}^N m_i \mathbf{x}_i \cdot \mathbf{x}_i \quad ,$$

where as usual $\mathbf{x}_i \cdot \mathbf{x}_i$ denotes scalar product.

a) Show that

$$\frac{1}{2} \frac{d^2 I}{dt^2} = \sum_{i=1}^N m_i \dot{\mathbf{x}}_i \cdot \dot{\mathbf{x}}_i + \sum_{i=1}^N m_i \mathbf{x}_i \cdot \ddot{\mathbf{x}}_i \quad .$$

[4 marks]

b) Give an expression for T , the total kinetic energy of the system.

[2 marks]

c) Give an expression for the gravitational force on star i due to star j in terms of vectors \mathbf{x}_i , \mathbf{x}_j . Hence write down an expression for the acceleration $\ddot{\mathbf{x}}_i$ of star i as a sum over $j \neq i$.

[4 marks]

d) Hence, prove that

$$\sum_{i=1}^N m_i \mathbf{x}_i \cdot \ddot{\mathbf{x}}_i = -\frac{1}{2} \sum_{i,j,(i \neq j)} \frac{G m_i m_j}{|\mathbf{x}_i - \mathbf{x}_j|} \quad ;$$

thus deduce the virial theorem,

$$2\langle T \rangle + \langle U \rangle = 0 \quad ,$$

where U is the total gravitational potential energy.

[6 marks]

e) The dark matter halo of our Galaxy has been modelled by a sphere with density profile

$$\rho(r) = \frac{\rho_0 a^2}{r^2 + a^2} \quad ,$$

where r is Galactocentric radius, ρ_0 is the halo central density and $a = 5$ kpc is the core radius. Observations indicate that the circular velocity of the Sun around the Galaxy is approximately 220 km s^{-1} , and the dark halo contributes approximately half of the total mass inside the Sun's orbit. Hence estimate the local density of dark matter ρ at $r = R_0 = 8$ kpc, in units of $M_\odot \text{ pc}^{-3}$.

You may use the standard integral $\int \frac{x^2}{x^2 + a^2} dx = x - a \tan^{-1} \frac{x}{a} + \text{const} \quad .$

[9 marks]

Turn over

Question B2

- a) The continuity equation for the distribution function f of stars in the six-parameter phase space $(x_1, x_2, x_3, v_1, v_2, v_3)$ of position \mathbf{x} and velocity \mathbf{v} states that

$$\frac{\partial f}{\partial t} + \sum_{j=1}^3 \left(\frac{\partial}{\partial x_j} \left(f \frac{dx_j}{dt} \right) + \frac{\partial}{\partial v_j} \left(f \frac{dv_j}{dt} \right) \right) = 0 ,$$

where t is time.

Derive the collisionless Boltzmann equation

$$\frac{\partial f}{\partial t} + \sum_{j=1}^3 \left(\frac{dx_j}{dt} \frac{\partial f}{\partial x_j} + \frac{dv_j}{dt} \frac{\partial f}{\partial v_j} \right) = 0$$

from the continuity equation, justifying relevant assumptions.

[5 marks]

- b) Describe the advantages of the Jeans equations relative to the collisionless Boltzmann equation for describing the distribution of stars in observed galaxies.

[2 marks]

- c) Derive the first of the Jeans equations,

$$\frac{\partial n}{\partial t} + \sum_{i=1}^3 \frac{\partial}{\partial x_i} (n \langle v_i \rangle) = 0 ,$$

from the collisionless Boltzmann equation, where n is the number density of stars and $\langle v_i \rangle$ is the mean value of the v_i velocity component at a point. (Explain your working and assumptions).

[8 marks]

- d) The velocity dispersion tensor σ_{ij} is defined by

$$\sigma_{ij}^2 = \frac{1}{n} \int (v_i - \langle v_i \rangle) (v_j - \langle v_j \rangle) f d^3\mathbf{v} ,$$

where v_1, v_2 and v_3 are the components of the velocity vector \mathbf{v} , n is the number density of stars in space, f is the distribution function, and i and $j = 1, 2$ and 3 . Prove that

$$\sigma_{ij}^2 = \langle v_i v_j \rangle - \langle v_i \rangle \langle v_j \rangle .$$

[4 marks]

- e) The third Jeans equation may be written as

$$n \frac{\partial \langle v_i \rangle}{\partial t} + n \sum_{j=1}^3 \langle v_j \rangle \frac{\partial \langle v_i \rangle}{\partial x_j} = -n \frac{\partial \Phi}{\partial x_i} - \sum_{j=1}^3 \frac{\partial}{\partial x_j} (n \sigma_{ij}^2) .$$

Give a physical interpretation of (a) the sum of the left-hand side, (b) the first term on the right-hand side, and (c) the second term on the right-hand side.

[6 marks]

Question B3

- a) Define the symbols $H I$, $H II$ and H_2 referring to hydrogen in the interstellar medium. For each of these species, describe (a) the typical environments in which they are found; (b) the main processes by which they are observed, including the physical emission/absorption processes and the key wavelengths.

[6 marks]

- b) Describe the main processes determining the relative abundance of each state of hydrogen above.

[3 marks]

- c) The thermal energy of an atom with temperature T is $\sim k_B T$, where k_B is Boltzmann's constant. A warm atomic gas cloud has a temperature of $5000K$. By considering the energy levels of a hydrogen atom, explain why the cloud does not produce a significant amount of Balmer line emission, unless it contains hot stars.

[2 marks]

- d) In a region of the Galaxy, the total mass of stars is M_{stars} , the total mass of interstellar gas is M_{gas} , and the mass of heavy elements in the interstellar medium is M_{metals} , while the metallicity of the gas is Z . The changes in these quantities in a small time interval are δM_{stars} , δM_{gas} , δM_{metals} and δZ respectively.

For the Simple Model of galactic chemical enrichment, derive the expression

$$\delta Z = \frac{\delta M_{\text{metals}}}{M_{\text{gas}}} - Z \frac{\delta M_{\text{gas}}}{M_{\text{gas}}} .$$

[4 marks]

- e) If δM_{metals} and δM_{stars} above are related by $\delta M_{\text{metals}} = -Z \delta M_{\text{stars}} + p \delta M_{\text{stars}}$, where p is the yield of heavy elements, show that

$$\delta Z = -p \frac{\delta M_{\text{gas}}}{M_{\text{gas}}} .$$

[2 marks]

- f) Suppose the Simple Model is modified to allow metal-free gas to accrete into the system (but nothing leaves), so M_{tot} can increase. Show that δZ is now given by

$$\delta Z = \frac{(p - Z) \delta M_{\text{tot}} - p \delta M_{\text{gas}}}{M_{\text{gas}}} .$$

In the simple case, where the gas accretion rate equals the rate at which mass is locked up into stars and stellar remnants, show that Z asymptotes to p . Explain physically why this is expected.

[8 marks]

Turn over

Question B4

- a) The fundamental plane for elliptical galaxies has the form

$$R_S I_0^{0.8} \sigma_0^{-1.3} = \text{constant} ,$$

where R_S is a characteristic radius, I_0 is central surface brightness and σ_0 is line-of-sight velocity dispersion. Show that this implies luminosity $L \propto \sigma_0^{2.6} / I_0^{0.6}$.

[3 marks]

- b) Explain how the above quantities are determined observationally, and used to estimate relative distances to galaxy clusters.

[4 marks]

- c) Explain the difference between “pressure support” and “rotational support” for a galaxy. Which of these dominates for spiral and elliptical galaxies respectively ?

[4 marks]

- d) What is meant by the terms *local standard of rest* and *asymmetric drift*. Describe how the *asymmetric drift* varies with stellar age for stars in the Solar neighbourhood. How does the velocity dispersion of stars depend on age? What effect might giant molecular clouds in the Galactic disc have had on the velocity dispersions of old stars?

[5 marks]

- e) The disk of the Milky Way may be modelled as a double-exponential disk with stellar density given by

$$\rho(R, z) = \rho_0 e^{-|z|/h} e^{-(R-R_0)/L} ,$$

where R, z are Galactocentric cylindrical coordinates, ρ_0 is the local stellar density, h is the scale height and L is the scale length. Observations imply $\rho_0 \simeq 0.1 M_\odot \text{pc}^{-3}$, $h \simeq 250 \text{pc}$, $L \simeq 3 \text{kpc}$ and $R_0 \simeq 8 \text{kpc}$. Calculate the total stellar mass of the disk in this model.

Briefly describe the main sources of uncertainty when estimating the disk stellar mass from this model.

[9 marks]

End of Paper - An Appendix of 1 page follows

Appendix - Useful Information

In this paper, π and e represent the standard mathematical constants.

G is the gravitational constant, with $G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$.

c is the speed of light, with $c = 3.00 \times 10^8 \text{ m s}^{-1}$.

The Boltzmann constant is $k_B = 1.38 \times 10^{-23} \text{ J K}^{-1}$.

One electron-volt (eV) = $1.6 \times 10^{-19} \text{ J}$.

1 parsec (pc) = $3.09 \times 10^{16} \text{ m}$.

1 astronomical unit (AU) = $1.50 \times 10^{11} \text{ m}$.

The mass of the Sun is $M_\odot = 2.0 \times 10^{30} \text{ kg}$.

The distance of the Sun from the Galactic Centre is $R_0 = 8.0 \text{ kpc}$.

Poisson's equation states that $\nabla^2\Phi = 4\pi G\rho$ at any point in a gravitational field, where Φ is the gravitational potential, G is the constant of gravitation, and ρ is the mass density at that point.

The Laplacian of a scalar function Φ in a spherical coordinate system (r, θ, ϕ) is :

$$\nabla^2\Phi = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial\Phi}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial\theta} \left(\sin\theta \frac{\partial\Phi}{\partial\theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2\Phi}{\partial\phi^2} .$$

The Laplacian of a scalar function Φ in a cylindrical coordinate system (R, ϕ, z) is :

$$\nabla^2\Phi = \frac{1}{R} \frac{\partial}{\partial R} \left(R \frac{\partial\Phi}{\partial R} \right) + \frac{1}{R^2} \frac{\partial^2\Phi}{\partial\phi^2} + \frac{\partial^2\Phi}{\partial z^2} .$$

The Jeans equations in a steady-state (i.e. time-independent), spherically-symmetric galaxy give the following result :

$$\frac{d}{dr} \left(n \langle v_r^2 \rangle \right) + \frac{n}{r} \left[2 \langle v_r^2 \rangle - \langle v_\theta^2 \rangle - \langle v_\phi^2 \rangle \right] = -n \frac{d\Phi}{dr} ,$$

in spherical coordinates, where n is the number density of stars at a distance r from the centre, v_r , v_θ and v_ϕ are the components of the velocity in the r , θ and ϕ directions, and $\Phi(r)$ is the gravitational potential.

In the absence of cosmological effects, the apparent magnitude m of an astronomical object in a photometric band is related to its absolute magnitude M in that band and its distance D from the observer by

$$m - M = 5 \log_{10}(D/\text{pc}) - 5 + A ,$$

where A is the extinction in the same band expressed in magnitudes.

End of Appendix