SPA7010U/SPA7010P: THE GALAXY

Solutions for Coursework 3

Questions distributed on: 12 March 2020.

Solution 3.1 [**Assessed question**]

(a) The observed colour indices of an object that is affected by reddening by dust are (U – B) and (B – V).

The intrinsic values (i.e. what would be observed if no dust were present) are $(\mathrm{U}-\mathrm{B})_0$ and $(\mathrm{B}-\mathrm{V})_0$.

The colour excesses, which measure the extent of the reddening by dust, are therefore $E_{\mathrm{U-B}} = (\mathrm{U}-\mathrm{B}) - (\mathrm{U}-\mathrm{B})_0$ and $E_{\mathrm{B-V}} = (\mathrm{B}-\mathrm{V}) - (\mathrm{B}-\mathrm{V})_0$.

So the observed values are $(\mathrm{U}-\mathrm{B}) = (\mathrm{U}-\mathrm{B})_0 + E_{\mathrm{U-B}}$ and $(\mathrm{B}-\mathrm{V}) = (\mathrm{B}-\mathrm{V})_0 + E_{\mathrm{B-V}}$.

Substituting for these into the expression $Q \equiv (\mathrm{U}-\mathrm{B}) - (E_{\mathrm{U-B}}/E_{\mathrm{B-V}})(\mathrm{B}-\mathrm{V})$,

$$
\begin{aligned}
Q &= (\mathrm{U}-\mathrm{B})_0 + E_{\mathrm{U-B}} - \frac{E_{\mathrm{U-B}}}{E_{\mathrm{B-V}}}\left((\mathrm{B}-\mathrm{V})_0 + E_{\mathrm{B-V}}\right) \\
&= (\mathrm{U}-\mathrm{B})_0 + E_{\mathrm{U-B}} - \frac{E_{\mathrm{U-B}}}{E_{\mathrm{B-V}}}(\mathrm{B}-\mathrm{V})_0 - E_{\mathrm{U-B}} \\
&= (\mathrm{U}-\mathrm{B})_0 - \frac{E_{\mathrm{U-B}}}{E_{\mathrm{B-V}}}(\mathrm{B}-\mathrm{V})_0
\end{aligned}
$$

So assuming $E_{\mathrm{U-B}}/E_{\mathrm{B-V}}$ is constant (as stated in the question), the Q parameter is equal to the value it would have in the absence of interstellar extinction i.e. Q is independent of the strength of the extinction.

(Note: the Q parameter is useful for hot young stars because it can be used to determine the spectral type, and hence surface temperature. However, it becomes less sensitive to spectral type (and therefore temperature) for older stars and is not used much for these.)

(b) Substituting the observed magnitudes for the hot star and the supplied value $E_{\mathrm{U-B}}/E_{\mathrm{B-V}} = 0.72$ into the expression for Q, we get $Q = -0.84$ mag.

(c) As we have established, Q is independent of reddening, so the intrinsic value of Q is also -0.84. So, from the table, $Q = -0.84$ corresponds to a spectral type B0V.

(d) The observed (B – V) colour index is (B – V) = 12.69 – 12.00 = 0.69 mag, while from the table, the intrinsic (B–V) index for this type of star is $(\mathrm{B}-\mathrm{V})_0 = -0.31$ mag.

The (B – V) colour excess is therefore $E_{\mathrm{B-V}} = (\mathrm{B}-\mathrm{V}) - (\mathrm{B}-\mathrm{V})_0 = 0.69 - (-0.31) =$ 1.00 mag.

(e) Assuming a reddening ratio $\frac{A_V}{E_{\mathrm{B-V}}} \sim 3.00$, the V-band extinction $A_V = 3.00\ E_{\mathrm{B-V}}$, giving $A_V = 3.00 \times 1.00 = 3.00$ mag.

(f) The standard formula $m - M = 5\log_{10}(D/\mathrm{pc}) - 5 + A$ provides the relationship between the observed apparent magnitude m, absolute magnitude M, distance D in parsecs, and extinction A (see Lecture 1). For the V band we get

$$\mathrm{V} - M_{\mathrm{V}} = 5\log_{10}(D/\mathrm{pc}) - 5 + A_{\mathrm{V}}$$

$$\therefore\quad \log_{10}(D/\mathrm{pc}) \;=\; \frac{1}{5}\big(5 + \mathrm{V} - M_{\mathrm{V}} - A_{\mathrm{V}}\big) \;=\; \frac{1}{5}(5 + 12.00 - (-4.10) - 3.00) \;=\; 3.62$$

using $M_{\mathrm{V}} = -4.1$ for this star (from the table). So the distance is $D = 10^{3.62}$ pc = 4169 pc i.e. 4.17 kpc.

(g) If the longitude of the star is 180° and it lies in the Galactic plane, its distance from the Galactic Centre is $R = R_0 + 4.17$ kpc = 12.17 kpc, assuming $R_0 = 8.0$ kpc is the approximate distance of the Sun from the Galactic centre.

[Total 50 marks]

End of assessed question

Solution 3.2

(a) Ultraviolet radiation from hot background stars ionises the gas, producing the H II region. The hydrogen ions and free electrons then recombine, emitting photons. Atoms/ions in excited energy levels (i.e. above the ground state) decay radiatively, cascading down through energy levels, emitting line photons with particular energies. This is observed as strong emission lines from the excited gas. The hydrogen Balmer transitions (down to the $n = 2$ level) are prominent in optical spectra. Lyman transitions (down to the ground state, $n = 1$) produce ultraviolet photons which are absorbed again, so ultraviolet radiation is not observed. Some ions can also be collisionally excited.

(b) The 21cm radio line is produced by hyperfine transitions in atomic hydrogen between parallel and anti-parallel spin states caused by the coupling of the spins of the proton and electron in the hydrogen atom.

This line cannot be observed directly in laboratory spectra because the transition probability is so low that interactions between atoms interfere before appreciable emission/absorption from the 21cm transition can occur in the laboratory.

(c) The [O/Fe] parameter is $[\mathrm{O/Fe}] = \log(1/10) = -1.00$.

(d) The surface mass density of stars can be obtained by integrating the density over height z:

$$\Sigma_s = \int_{-\infty}^{\infty} \rho_s(z)\,\mathrm{d}z = \int_{-\infty}^{\infty} \rho_{so}\,\mathrm{e}^{-|z|/h_s}\,\mathrm{d}z = 2\int_0^{\infty} \rho_{so}\,\mathrm{e}^{-z/h_s}\,\mathrm{d}z \quad \text{(from symmetry)}$$

$$= 2\,\rho_{so}\int_0^{\infty} \mathrm{e}^{-z/h_s}\,\mathrm{d}z = 2\,\rho_{so}\left[-h_s\,\mathrm{e}^{-z/h_s}\right]_{z=0}^{\infty} = 2\,\rho_{so}\,h_s\ .$$

Similarly, for the gas, $\Sigma_g = 2\,\rho_{go}\,h_g$. Therefore,

$$\frac{\Sigma_s}{\Sigma_g} = \frac{2\,\rho_{so}\,h_s}{2\,\rho_{go}\,h_g} = \frac{\rho_{so}}{\rho_{go}}\,\frac{h_s}{h_g} = 6 \times \frac{250}{150} = 10\ .$$

So $\Sigma_s = 10\,\Sigma_g$ at the Sun's distance from the Galactic Centre.

(e) Dust density ρ_d closely follows that of gas and observations show that $\rho_d/\rho_g \simeq 0.1$. Therefore we expect $\Sigma_s \simeq 100\,\Sigma_d$ at the Sun's distance from the Galactic Centre.

Solution 3.3

(a) (i) Most of the helium originates from big bang nucleosynthesis, while (ii) most the heavy elements were formed in evolving stars and ejected into the interstellar medium through interstellar winds and supernovae.

(b) The metallicity of the interstellar medium is $Z = -p \ln \mu$ in the Simple Model, where p is the yield (a constant) and μ us the gas fraction (the fraction of the total baryonic mass in the form of gas) at any time.

According to this expression, $\mu \to 0$, $Z \to \infty$. This is not physically realistic, given that from the definition of the heavy element mass fraction Z, material consisting entirely of heavy elements (no hydrogen or helium) would have $Z = 1$, the maximum value Z can take. Also, the $Z = -p \ln \mu$ prediction only gives $Z \simeq 1$ for extremely small gas fractions ($\mu \sim 10^{-22}$) which are not likely to be encountered in any real galaxy.

(Note: the formal failure of the Simple Model prediction is the result of the approximations made i.e. assuming that Z is small. In practice, $Z \ll 1$ always and the Simple Model equations work accurately, provided that the basic assumptions behind the Simple Model are valid. If we rederived the equations without these small Z approximations, the Simple Model would give $Z \to 1$ as $\mu \to 0$.)

(c) The gas fraction $\mu = M_{\rm gas}/M_{\rm gas}(0)$. But $M_{\rm stars} + M_{\rm gas} = M_{\rm total} = M_{\rm gas}(0)$, which gives, $M_{\rm gas} = M_{\rm gas}(0) - M_{\rm stars}$. Therefore, the gas fraction is

$$\mu \;=\; \frac{M_{\rm gas}(0) - M_{\rm stars}}{M_{\rm gas}(0)} \;=\; 1 - \frac{M_{\rm stars}}{M_{\rm gas}(0)} \; . \tag{1}$$

The $Z = -p \ln \mu$ result can therefore be rewritten as

$$Z \;=\; -p \,\ln\left(1 - \frac{M_{\rm stars}}{M_{\rm gas}(0)}\right) \; .$$

Using the hint given in the question, the mean metallicity is $\langle Z \rangle = \dfrac{\int_0^{M_{\rm stars}} Z \, {\rm d}M'_{\rm stars}}{\int_0^{M_{\rm stars}} {\rm d}M'_{\rm stars}}$.

[Continued ...]

Therefore,

$$\begin{aligned}
\langle Z \rangle &= \frac{1}{M_{\rm stars}} \int_0^{M_{\rm stars}} Z \, {\rm d}M'_{\rm stars} \;=\; \frac{1}{M_{\rm stars}} \int_0^{M_{\rm stars}} - p \ln\left(1 - \frac{M'_{\rm stars}}{M_{\rm gas}(0)}\right) {\rm d}M'_{\rm stars} \\
&= - \frac{p}{M_{\rm stars}} \left[\left(M'_{\rm stars} - M_{\rm gas}(0) \right) \ln\left(1 - \frac{M'_{\rm stars}}{M_{\rm gas}(0)}\right) \;-\; M'_{\rm stars} \right]_0^{M_{\rm stars}} \\
&= - \frac{p}{M_{\rm stars}} \left(\left(M_{\rm stars} - M_{\rm gas}(0) \right) \ln\left(1 - \frac{M_{\rm stars}}{M_{\rm gas}(0)}\right) \;-\; M_{\rm stars} \;-\; 0 \right) \\
&= p \left(\left(-1 + \frac{M_{\rm gas}(0)}{M_{\rm stars}} \right) \ln\left(1 - \frac{M_{\rm stars}}{M_{\rm gas}(0)}\right) \;+\; 1 \right) \quad ,
\end{aligned}$$

which gives finally,

$$\langle Z \rangle \;=\; p \left(\frac{M_{\rm gas}(0)}{M_{\rm stars}} - 1 \right) \ln\left(1 - \frac{M_{\rm stars}}{M_{\rm gas}(0)}\right) \;+\; p \qquad (3) \quad .$$

(d) Rearranging equation (1),

$$\frac{M_{\rm stars}}{M_{\rm gas}(0)} \;=\; 1 - \mu \; . \qquad \therefore \quad \frac{M_{\rm gas}(0)}{M_{\rm stars}} \;=\; \frac{1}{1-\mu} \; . \qquad (2)$$

Substituting equations (1) and (2) into (3):

$$\langle Z \rangle \;=\; p \left(\frac{1}{1-\mu} - 1 \right) \ln \mu \;+\; p \;=\; p \left(\frac{1-(1-\mu)}{1-\mu} \right) \ln \mu \;+\; p \;=\; p \left(1 + \frac{\mu \ln \mu}{1-\mu} \right) \; ,$$

the required result.

(e) As gas is exhausted in star formation, $\mu \to 0$, $\mu \ln \mu/(1-\mu) \to 0$. Therefore, $\langle Z \rangle \to p$, the yield.

(Note: this is an interesting and important prediction of the Simple Model. In practice, $\mu > 0$ and therefore we expect the mean metallicity in a population of long-lived stars to be less than the yield.)