SPA7010U/SPA7010P: THE GALAXY

Coursework 2

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Question 2.1 [**Assessed question**]

The Plummer model has a gravitational potential $\Phi(r)$ at a distance r from its centre given by

$$\Phi(r) \;=\; -\,\frac{GM_{tot}}{\sqrt{r^2+a^2}}\;,$$

where M_{tot} is the total mass, a is a constant and G is the constant of gravitation.

Assuming that the velocity dispersion σ of a population of stars is isotropic and constant over the whole galaxy, and that there is no net rotation, show that the number density of these stars in this potential is

$$n(r) \;=\; n_0\;\exp\left[\frac{GM_{tot}}{a\,\sigma^2}\left(\frac{1}{\sqrt{1+r^2/a^2}}\;-\;1\right)\right]\;,$$

where $n_0 = n(0)$.

Hint:

The second Jeans equation in a spherically-symmetric potential may be expressed as

$$\frac{\mathrm{d}}{\mathrm{d}r}\Big(n\,\langle v_r^2\rangle\Big) \;+\; \frac{n}{r}\,\Big[\,2\langle v_r^2\rangle \;-\;\langle v_\theta^2\rangle\;-\;\langle v_\phi^2\rangle\Big] \;=\; -\,n\,\frac{\mathrm{d}\Phi}{\mathrm{d}r}\;,$$

for a spherical polar coordinate system (r, θ, ϕ), where $n(r)$ is the number density of stars in space, $\Phi(r)$ is the gravitational potential, and v_r, v_θ and v_z are the components of the velocity in the r, θ and ϕ directions at a point.

[Total 50 marks]

End of assessed question

Continued ...

Question 2.2

Using the expression for the gravitational potential $\Phi(r)$ given for the Plummer model, above

(a) Show that the mass $M(r)$ interior to a radius r is

$$M(r) \quad = \quad \frac{M_{tot}\, r^3}{(r^2 + a^2)^{\frac{3}{2}}}$$

(b) Show that the density $\rho(r)$ at a radius r is

$$\rho(r) \quad = \quad \frac{3M_{tot}}{4\pi} \, \frac{a^2}{(r^2 + a^2)^{\frac{5}{2}}}$$

(c) Comment on the effect of the constant, a, on the behaviour of the model.

Question 2.3

A galaxy is modelled using a spherically-symmetric gravitational potential of the form

$$\Phi(r) \quad = \quad - \, \frac{4\pi G k}{r} \, \ln\left(\frac{r + a}{a}\right) \quad ,$$

where r is the radial distance from the centre of the galaxy, a and k are constants and G is the constant of gravitation.

Using Poisson's equation $\nabla^2\Phi = 4\pi G\rho$, show that the mass density ρ as a function of distance r implied by this potential is

$$\rho(r) \quad = \quad \frac{k}{r(r + a)^2} \quad .$$