SPA7010U/SPA7010P: THE GALAXY

Coursework 1

Distributed on: 6 February 2020. *To be submitted by:* 20 February 2020

Question 1.1 [**Assessed question**]

(a) An astronomer performs optical spectroscopy on a faint galaxy. The spectrum shows a strong continuum with absorption lines and some emission lines superimposed. Based on this information, which morphological type would you assign to the the galaxy (i.e. is it likely to be spiral, elliptical or irregular)? Briefly justify your answer. [4 marks]

(b) The astronomer observes a second galaxy and finds very strong emission lines superimposed on a continuum, and some absorption lines. How does the star formation rate of this second galaxy compare with that in the first? What morphological type might the second galaxy be (i.e. spiral, elliptical or irregular)? [4 marks]

(c) What is the difference between collisional and collisionless interactions between two massive bodies? Are the interactions between two gas clouds collisional or collisionless? [4 marks]

(d) Two small systems of stars collide. In this particular case, the gravitational field of any one star changes the motions of other nearby stars substantially. Is the interaction between the two systems collisional or collisionless? How would this compare with the stars in two galaxies that merge together? [4 marks]

(e) A large stationary gas cloud collapses under its own gravitation. If 20 % of the initial potential energy is converted into heat, raising the temperature of the gas, is this collapse dissipative or dissipationless? [4 marks]

(f) Suppose some category of galaxies has an observed surface brightness profile $I(R) = I_0\, f(R/R_0)$ with all galaxies having the same I_0 and function f, but different galaxies having different R_0. If the mass-to-light ratio, M/L, is constant everywhere, show that

$$L \propto v^4$$

where L is the total luminosity and v is a characteristic velocity. Briefly state how this compares to the actual observed relationship between luminosity and velocity for spiral and elliptical galaxies, respectively. [30 marks]

[Total 50 marks]

End of assessed question

Question 1.2

A family of radial density profiles $\rho(r)$ that have been popular for the theoretical modelling of spherically symmetric galaxies are called Dehnen models. These have density profiles that vary with radius as follows:

$$\rho(r) \quad = \quad \frac{q\,a}{4\pi}\,\frac{r^q}{r^3\,(r+a)^{q+1}}\quad M_{tot} \ ,$$

where r is the radial distance from the centre of the galaxy, q is an adjustable parameter, a is a scaling constant (determining the size of the galaxy), and M_{tot} is the total mass. The special case of $q = 1$, which is called the Jaffe model, is particularly important because it is found to fit the observed $I(R)$ of ellipticals at least as well as the de Vaucouleurs $R^{1/4}$ profile.

(a) What is the mass $M(r)$ interior to a radius r for any value of q?

(b) What is the gravitational potential of a mass distribution having a Jaffe $\rho(r)$?

(c) The Dehnen models have an interesting limit as $q \to 0$. What is it?

You may use the standard integral

$$\int \frac{r^{q-1}}{(r+a)^{q+1}}\,\mathrm{d}r \quad = \quad \frac{1}{q\,a}\,\frac{r^q}{(r+a)^q} \ + \ \text{constant} \ .$$

Question 1.3

The Navarro-Frenk-White density profile is often used to represent galaxies. In this profile the density ρ at a distance r from the centre of the galaxy is given by

$$\rho(r) \quad = \quad \frac{k}{r(r+a)^2} \ ,$$

where k and a are constants.

(a) What is the mass $M(r)$ interior to a radius r implied by this profile?

(b) What happens to $M(r)$ as $r \to \infty$?

(c) What is the gravitational potential Φ at a radius r?

(d) What is the central density implied by this profile? Is it physically realistic?

Continued ...

Hints for Questions 1.2 and 1.3

These questions involve calculations relating to spherically symmetric potentials. Under spherical symmetry, the gravitational potential Φ, the mass interior to a radius and the density ρ are all functions of the radius r from the centre alone.

Converting between $\Phi(r)$ and the density $\rho(r)$ can be done using Poisson's equation

$$\nabla^2\Phi = 4\pi G\rho$$

(a general result for any gravitational field and any distribution of mass), where G is the constant of gravitation.

In the case of spherical symmetry, $\nabla^2\Phi$ is a function of r alone (see Appendix C) with

$$\nabla^2\Phi = \frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}\Phi}{\mathrm{d}r}\right)$$

(because $\partial\Phi/\partial\theta$ and $\partial\Phi/\partial\phi$ are zero for spherical symmetry).

Also, for spherical symmetry, the following relations exist between the gravitational potential $\Phi(r)$, the mass $M(r)$ within a radius r, and density $\rho(r)$:

$$\frac{\mathrm{d}M}{\mathrm{d}r} = 4\pi r^2\rho \qquad \text{and} \qquad M(r) = \frac{r^2}{G}\frac{\mathrm{d}\Phi}{\mathrm{d}r}\ .$$

The first of these two equations is just the equation of conservation of mass.

We can therefore convert from $\Phi(r)$ to $M(r)$ and to $\rho(r)$ by differentiation, and from $\rho(r)$ to $M(r)$ and to $\Phi(r)$ by integration.