

MSc/MSci Examination

Main Examination Period 2017

SPA7010P / SPA7010U

The Galaxy

Duration: 2 hours 30 minutes

YOU ARE NOT PERMITTED TO READ THE CONTENTS OF THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR.

Instructions:

Answer ALL questions from Section A. Answer ONLY TWO questions from Section B. Section A carries 50 marks, each question in section B carries 25 marks.

If you answer more questions than specified, only the first answers (up to the specified number) will be marked. Cross out any answers that you do not wish to be marked.

Only non-programmable calculators are permitted in this examination. Please state on your answer book the name and type of machine used.

Complete all rough workings in the answer book and cross through any work that is not to be assessed.

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Examiners:

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Dr. D. Tsiklauri

SECTION A**Answer ALL questions in Section A****Question A1**

Sketch the “Hubble tuning fork diagram” which illustrates the morphological classification of normal galaxies, and explain how galaxy properties (colours, spectra, and gas and dust content) vary across the diagram.

[6 marks]**Question A2**

Explain the difference between collisional and collisionless processes in galaxy formation. In an encounter between two galaxies, explain which of these applies to the stars and gas respectively.

[4 marks]**Question A3**

Explain the difference between “pressure support” and “rotational support” for a galaxy. Which of these dominates for spiral and elliptical galaxies respectively ?

[4 marks]**Question A4**

State (without proof) the virial theorem for a self-gravitating stellar system, defining the terms used. What are the conditions required for its application?

A cluster of galaxies has an observed radius of 900 kpc and a velocity dispersion of 750 km s^{-1} . Use the virial theorem to estimate the total mass, giving your answer in solar masses.

(You may assume the potential energy of a uniform sphere of mass M and radius R is $U = -3GM^2/(5R)$).

[6 marks]**Question A5**

Define the term “integral of motion” for a function of position and velocity in a galaxy. State (without proof) two integrals of motion for a galaxy with a time-independent and axisymmetric potential.

[4 marks]

Question A6

Explain the meaning of the terms HI , HII and H_2 referring to hydrogen in the Galaxy. State the main processes which convert HI into HII and H_2 respectively.

[5 marks]**Question A7**

In Galactic chemical evolution, give the standard definition of the symbols X , Y , Z , and quote typical values for these in the local interstellar medium. State what processes respectively produced (a) most of the helium, and (b) most of the heavy elements.

[4 marks]**Question A8**

A star near the Galactic plane is observed to have apparent magnitudes in the blue and visual bands of $B = 18.39$, $V = 17.44$. Spectroscopy indicates that it has a Sun-like spectrum with absolute magnitudes $M_B = 5.48$, $M_V = 4.83$ in these bands. Assuming that the reddening ratio for interstellar dust is $A_V/E(B - V) = 3.0$, estimate (i) the reddening of the star, (ii) the V-band extinction in magnitudes, and (iii) the distance to the star.

[5 marks]**Question A9**

For a gravitational lens with perfect alignment, the angular Einstein ring radius is given by

$$\theta_E = \sqrt{\frac{4GM}{c^2} \frac{D_{LS}}{D_S D_L}} \text{ rad},$$

where M is the lens mass, D_L is the distance from Earth to the lens, D_S is the distance from Earth to the source, and D_{LS} is the distance between lens and source.

Assuming a source star in the Galactic Bulge at $D_S = 8 \text{ kpc}$, and a lens of mass $0.5 M_\odot$ at distance 6 kpc , evaluate θ_E in arcseconds; comment on the implications for observations of microlensing.

[6 marks]**Question A10**

Describe how observations of the Milky Way and Local Group galaxies can be used to constrain the location and nature of dark matter.

[6 marks]

SECTION B

Answer TWO questions from Section B

Question B1

- a) Define the terms *weak encounter* and *strong encounter* for two stars approaching each other in a large stellar system.

[2 marks]

- b) In a weak encounter between two stars each of mass m with relative velocity v , the change in the velocity of one star in the reference frame of the other is given by

$$\delta v = \frac{2Gm}{bv},$$

where G is the constant of gravitation and b is the impact parameter.

A star moves through a spherical region of overall radius R containing N stars distributed uniformly in space. If the mean change in the square of the velocity is $\delta(v^2) = (\delta v)^2$ in a single weak encounter, show that the changes in v^2 caused by weak encounters with impact parameters in the range b to $b + db$ during a time t is

$$\Delta v^2 = \left(\frac{2Gm}{bv}\right)^2 \left(\frac{3bvtNdb}{2R^3}\right),$$

and hence show that the total change in v^2 in a time t caused by weak encounters with all impact parameters is

$$\Delta v^2(t) = 6 \left(\frac{Gm}{v}\right)^2 \frac{vtN}{R^3} \ln\left(\frac{b_{\max}}{b_{\min}}\right),$$

where b_{\max} and b_{\min} are the largest and smallest values of the impact parameter.

[7 marks]

- c) From the result above, write down an expression for the relaxation time T_{relax} ; hence show that for suitable choices of b_{\min} and b_{\max} , the ratio of the relaxation time to the crossing time, T_{cross} , is given approximately by

$$\frac{T_{\text{relax}}}{T_{\text{cross}}} \approx \frac{N}{6 \ln N}.$$

(You may assume that in a stellar system of radius R containing N stars each of mass m , the typical velocity v is given by $v \approx \sqrt{GNm/R}$.)

[7 marks]

- d) The dark matter halo of our Galaxy has been modelled by a spherical halo with density profile

$$\rho(r) = \frac{\rho_0 a^2}{r^2 + a^2}$$

where r is the Galactocentric radius, ρ_0 is the halo central density and $a = 5$ kpc is the core radius. Observations indicate that the circular velocity of the Sun around the Galaxy is approximately 220 km s^{-1} , and the dark halo contributes approximately half of the total mass inside the Sun's orbit.

Hence estimate the local density of dark matter ρ at $r = R_0 = 8$ kpc, in units of $M_\odot \text{ pc}^{-3}$.

(You may quote the standard integral $\int \frac{x^2}{x^2 + a^2} dx = x - a \tan^{-1} \frac{x}{a} + \text{const.}$.)

[9 marks]

Question B2

a) The collisionless Boltzmann equation states that

$$\frac{\partial f}{\partial t} + \sum_{j=1}^3 \left(\frac{dx_j}{dt} \frac{\partial f}{\partial x_j} + \frac{dv_j}{dt} \frac{\partial f}{\partial v_j} \right) = 0 ,$$

where $f(\mathbf{x}, \mathbf{v}, t)$ is the distribution function, x_j and v_j are the j -components of position and velocity, and t is time.

Derive from this the second of the Jeans equations,

$$\frac{\partial(n\langle v_i \rangle)}{\partial t} + \sum_{j=1}^3 \frac{\partial}{\partial x_j} (n\langle v_i v_j \rangle) = - \frac{\partial \Phi}{\partial x_i} n ,$$

where n is the number density of stars, $\langle v_i \rangle$ is the mean value of the v_i velocity component at a point, and $\Phi(\mathbf{x}, t)$ is the gravitational potential. (Explain your definitions, working and assumptions).

[10 marks]

b) One of the Jeans equations in a cylindrical coordinate system (R, θ, z) with the origin at the centre of the Galaxy, and $z = 0$ in the Galactic plane, gives

$$\frac{\partial(n\langle v_z \rangle)}{\partial t} + \frac{\partial(n\langle v_R v_z \rangle)}{\partial R} + \frac{\partial(n\langle v_z^2 \rangle)}{\partial z} + \frac{n\langle v_R v_z \rangle}{R} = -n \frac{\partial \Phi}{\partial z} ,$$

where n is the star number density, v_R and v_z are the velocity components in the R and z directions, $\Phi(R, z)$ is the Galactic gravitational potential and t is time. Assuming that the Galaxy is in a steady state, show that the surface mass density $\Sigma(z, R_0)$ within a distance z of the mid-plane of the Galactic disc at the solar radius R_0 is given by

$$\Sigma(z, R_0) \simeq \frac{-1}{2\pi G n} \frac{\partial}{\partial z} (n\langle v_z^2 \rangle)$$

for stars in the direction of the Galactic poles. Explain the assumptions you make.

[7 marks]

c) The first Jeans equation is given by

$$\frac{\partial n}{\partial t} + \sum_{j=1}^3 \frac{\partial}{\partial x_j} (n\langle v_j \rangle) = 0 ,$$

with symbols defined as in part (a) above. The velocity dispersion tensor σ_{ij}^2 is defined by

$$\sigma_{ij}^2 = \langle v_i v_j \rangle - \langle v_i \rangle \langle v_j \rangle .$$

By a suitable substitution in the second Jeans equation and using also the first Jeans equation, derive the third Jeans equation:

$$n \frac{\partial \langle v_i \rangle}{\partial t} + n \sum_{j=1}^3 \langle v_j \rangle \frac{\partial \langle v_i \rangle}{\partial x_j} = -n \frac{\partial \Phi}{\partial x_i} - \sum_{j=1}^3 \frac{\partial}{\partial x_j} (n \sigma_{ij}^2) .$$

Give physical interpretations of (a) the sum of all terms on the left-hand side, (b) the first term on the right-hand side, and (c) the second term on the right-hand side.

[8 marks]

Question B3

- a) The distance l from the Milky Way to M31 (the Andromeda galaxy) is assumed to satisfy the differential equation

$$\frac{d^2l}{dt^2} = -\frac{GM}{l^2}$$

where t is time after the Big Bang, and the constant M is the total mass of the Local Group. Verify that

$$\begin{aligned} t &= \tau_0(\eta - \sin \eta), \\ l &= (GM\tau_0^2)^{\frac{1}{3}}(1 - \cos \eta) \end{aligned}$$

with τ_0 a constant, and η a parameter, is a solution of the above differential equation.

[8 marks]

- b) Explain how one may use observable quantities in the above equations to estimate the total mass M of the Local Group.

[3 marks]

- c) Describe the main observations which are used to constrain both the density profile and the nature of dark matter around our Galaxy. Also, discuss the main observational limitations or uncertainties in these methods.

[6 marks]

- d) The dark matter haloes of many galaxies can be well fitted by a “Burkert model” with a spherically-symmetric density profile given by

$$\rho(r) = \frac{\rho_0}{(1 + r/a)(1 + (r/a)^2)},$$

where r is galactocentric radius, ρ_0 is the central density, and a is a constant core radius. Prove that for this profile the enclosed mass is given by

$$M(r) = k \left[\ln(1 + r/a) + \frac{1}{2} \ln(1 + r^2/a^2) - \arctan(r/a) \right],$$

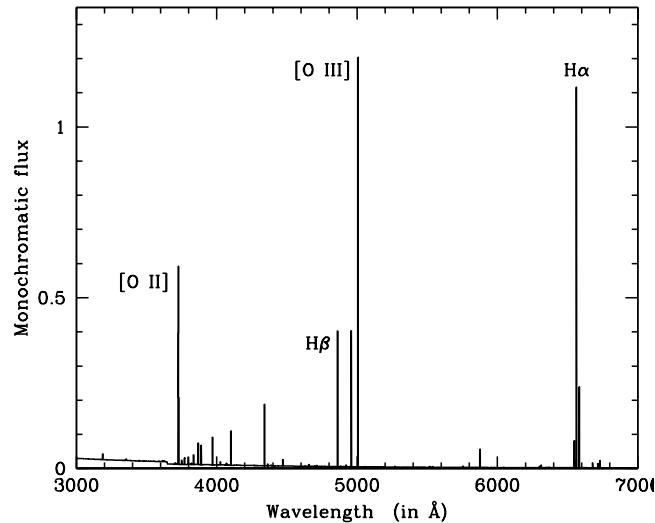
and derive the value of k in terms of ρ_0 and a .

Discuss the limiting behaviour of this mass profile at small radii $r \ll a$ and large radii $r \gg a$. (You may quote the partial fraction identity

$$\frac{r^2}{(a+r)(a^2+r^2)} \equiv \frac{1}{2} \left[\frac{1}{a+r} + \frac{(r-a)}{a^2+r^2} \right]. \quad)$$

[8 marks]

Question B4



- a) The figure (above) shows the visible-wavelength spectrum of the Orion Nebula. Briefly explain the different emission lines labelled, and the physical mechanisms responsible for producing them.

[3 marks]

- b) Explain, based on the energy levels of the hydrogen atom, why an HII region emits approximately one Balmer photon for each Lyman-continuum photon ($\lambda < 912\text{\AA}$) emitted by hot stars inside it.

[4 marks]

- c) Give a description of dust in the interstellar medium, with reference to its physical properties, its importance for star formation, its effects on observations and the methods used to observe it.

[6 marks]

- d) List any four of the five assumptions behind the *Simple Model* of galactic chemical evolution.

[4 marks]

- e) In the Simple Model of galactic chemical evolution, the change δZ in the heavy element mass fraction Z is given by

$$\delta Z = -p \frac{\delta M_{\text{gas}}}{M_{\text{gas}}},$$

where p is a parameter called the yield and M_{gas} is the mass of gas. Derive an expression for Z as a function of stellar mass $M_{\text{stars}}(t)$ at time t , and thence show that the mean metallicity of a population of long-lived stars is given by

$$\langle Z \rangle = p \left(\frac{M_{\text{gas}}(0)}{M_{\text{stars}}} - 1 \right) \ln \left(1 - \frac{M_{\text{stars}}}{M_{\text{gas}}(0)} \right) + p.$$

(You may quote the standard integral

$$\int \ln(1 - x/a) dx = (x - a) \ln(1 - x/a) - x + \text{constant} \quad).$$

[8 marks]

End of Paper - An Appendix of 1 page follows

Appendix - Useful Information

In this paper, π and e represent the standard mathematical constants.

G is the gravitational constant, with $G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$.

c is the velocity of light, with $c = 3.00 \times 10^8 \text{ m s}^{-1}$.

1 parsec (pc) = $3.09 \times 10^{16} \text{ m}$.

1 astronomical unit (AU) = $1.50 \times 10^{11} \text{ m}$.

The mass of the Sun is $M_{\odot} = 2.0 \times 10^{30} \text{ kg}$.

The distance of the Sun from the Galactic Centre is $R_0 = 8.0 \text{ kpc}$.

Poisson's equation states that $\nabla^2\Phi = 4\pi G\rho$ at any point in a gravitational field, where Φ is the gravitational potential, G is the constant of gravitation, and ρ is the mass density at that point.

The Laplacian of a scalar function Φ in a spherical coordinate system (r, θ, ϕ) is

$$\nabla^2\Phi = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial\Phi}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial\theta} \left(\sin\theta \frac{\partial\Phi}{\partial\theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2\Phi}{\partial\phi^2} .$$

The Laplacian of a scalar function Φ in a cylindrical coordinate system (R, ϕ, z) is

$$\nabla^2\Phi = \frac{1}{R} \frac{\partial}{\partial R} \left(R \frac{\partial\Phi}{\partial R} \right) + \frac{1}{R^2} \frac{\partial^2\Phi}{\partial\phi^2} + \frac{\partial^2\Phi}{\partial z^2} .$$

The Jeans equations in a steady-state (i.e. time-independent), spherically-symmetric galaxy give the following result

$$\frac{d}{dr} \left(n \langle v_r^2 \rangle \right) + \frac{n}{r} \left[2\langle v_r^2 \rangle - \langle v_{\theta}^2 \rangle - \langle v_{\phi}^2 \rangle \right] = -n \frac{d\Phi}{dr} ,$$

in spherical coordinates, where n is the number density of stars at a distance r from the centre, v_r , v_{θ} and v_{ϕ} are the components of the velocity in the r , θ and ϕ directions, and $\Phi(r)$ is the gravitational potential.

In the absence of cosmological effects, the apparent magnitude m of an astronomical object in a photometric band is related to its absolute magnitude M in that band and its distance D from the observer by

$$m - M = 5 \log_{10}(D/\text{pc}) - 5 + A ,$$

where A is the extinction in the same band expressed in magnitudes.

Oort's constants within the Galaxy are defined as

$$A \equiv \frac{1}{2} \left(\frac{\langle v_{\phi} \rangle}{R} - \frac{\partial \langle v_{\phi} \rangle}{\partial R} \right) \quad \text{and} \quad B \equiv -\frac{1}{2} \left(\frac{\langle v_{\phi} \rangle}{R} + \frac{\partial \langle v_{\phi} \rangle}{\partial R} \right) ,$$

where $\langle v_{\phi} \rangle$ is the mean tangential velocity in the Galactic disc, and R is the distance from the Galactic Centre, and the above are evaluated at $R = R_0$.

End of Appendix