Queen Mary
University of London

MSc/MSci Examination

12th May 2016 14:30 – 17:00

SPA7010P/SPA7010U/ASTM002 The Galaxy Duration: 2 hours 30 minutes

YOU ARE NOT PERMITTED TO READ THE CONTENTS OF THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR.

Instructions:

Answer ALL questions from Section A. Answer ONLY TWO questions from Section B. Section A carries 50 marks, each question in section B carries 25 marks.

If you answer more questions than specified, only the first answers (up to the specified number) will be marked. Cross out any answers that you do not wish to be marked.

Only non-programmable calculators are permitted in this examination. Please state on your answer book the name and type of machine used.

Complete all rough workings in the answer book and cross through any work that is not to be assessed.

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EXAM PAPERS MUST NOT BE REMOVED FROM THE EXAM ROOM.

Examiners:
Dr. W. Sutherland
Dr. J. Cho

Useful Information

In this paper, π and e represent the standard mathematical constants.
G is the gravitational constant, with $G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}.
c is the velocity of light, with $c = 3.00 \times 10^8$ m s^{-1}.
The Boltzmann constant is $k_B = 1.38 \times 10^{-23}$ J K^{-1} .
One electron-volt (eV) $= 1.6 \times 10^{-19}$ J.
1 parsec (pc) $= 3.09 \times 10^{16}$ m.
1 astronomical unit (AU) $= 1.50 \times 10^{11}$ m.
The mass of the Sun is $M_\odot = 2.0 \times 10^{30}$ kg.
The distance of the Sun from the Galactic Centre is $R_0 = 8.0$ kpc.

Poisson's equation states that $\nabla^2\Phi = 4\pi G\rho$ at any point in a gravitational field, where Φ is the gravitational potential, G is the constant of gravitation, and ρ is the mass density at that point.

The Laplacian of a scalar function Φ in a spherical coordinate system (r, θ, ϕ) is

$$\nabla^2\Phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2} .$$

The Laplacian of a scalar function Φ in a cylindrical coordinate system (R, ϕ, z) is

$$\nabla^2\Phi = \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\Phi}{\partial R}\right) + \frac{1}{R^2}\frac{\partial^2\Phi}{\partial\phi^2} + \frac{\partial^2\Phi}{\partial z^2} .$$

The Jeans equations in a steady-state (i.e. time-independent), spherically-symmetric galaxy give the following result

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(n\,\langle v_r^2\rangle\right) + \frac{n}{r}\left[2\langle v_r^2\rangle - \langle v_\theta^2\rangle - \langle v_\phi^2\rangle\right] = -\,n\,\frac{\mathrm{d}\Phi}{\mathrm{d}r} ,$$

in spherical coordinates, where n is the number density of stars at a distance r from the centre, v_r, v_θ and v_ϕ are the components of the velocity in the r, θ and ϕ directions, and $\Phi(r)$ is the gravitational potential.

In the absence of cosmological effects, the apparent magnitude m of an astronomical object in a photometric band is related to its absolute magnitude M in that band and its distance D from the observer by

$$m - M = 5\log_{10}(D/\mathrm{pc}) - 5 + A ,$$

where A is the extinction in the same band expressed in magnitudes.

SECTION A Answer ALL questions in Section A

Question A1

Describe the main observed characteristics of spiral galaxies, referring to morphology, colours, spectra, and gas and dust content; also discuss how these vary with Hubble type, from Sa to Sc or SBa to SBc.

[6 marks]

Question A2

The Tully-Fisher relation for spiral galaxies states that $L \simeq k v_{\text{rot}}^4$, where L is luminosity, v_{rot} is the peak rotational velocity and k is a constant.
One spiral galaxy in the Virgo cluster is observed with $v_{\text{rot}} = 180\,\text{km}\,\text{s}^{-1}$ and apparent magnitude $m_V = 12.0$. Another galaxy, in the Coma cluster, is observed with $v_{\text{rot}} = 240\,\text{km}\,\text{s}^{-1}$ and $m_V = 14.45$. Estimate the ratio of the distances to the two clusters.

[5 marks]

Question A3

Explain the difference between collisional and collisionless processes in galaxy formation. Explain why gas clouds in a galaxy tend to settle into a rotating disk.

[4 marks]

Question A4

Define the terms "strong encounter" and "weak encounter" in relation to an encounter between two stars in a large stellar system.
Show that in a weak encounter between two stars of mass m, the change δv in velocity v of one star in the reference frame of the other is given by

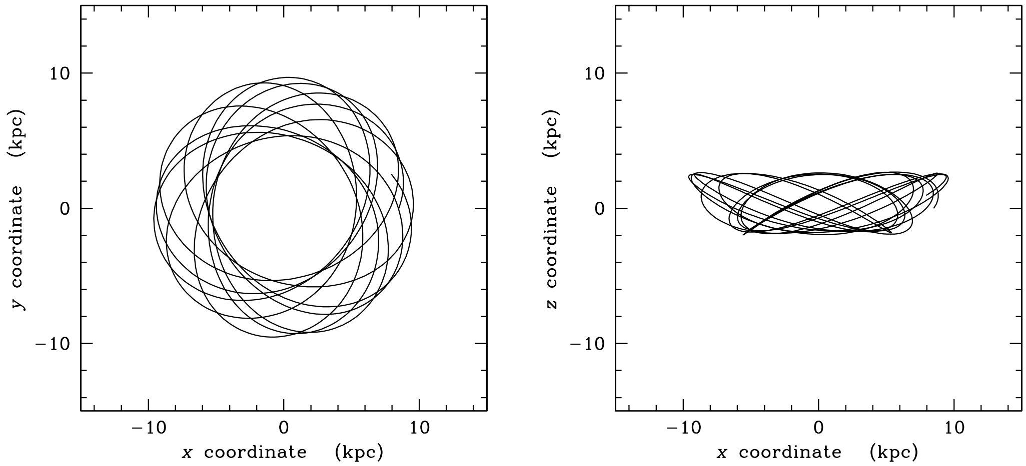
$$\delta v = \frac{2Gm}{bv}$$

where G is the gravitational constant and b is the impact parameter. (You may assume that the deflection angle is small, and you may quote the result $\int_{-\infty}^{\infty} (k_1 + k_2 s^2)^{-3/2}\, ds = 2/(k_1\sqrt{k_2})$) .

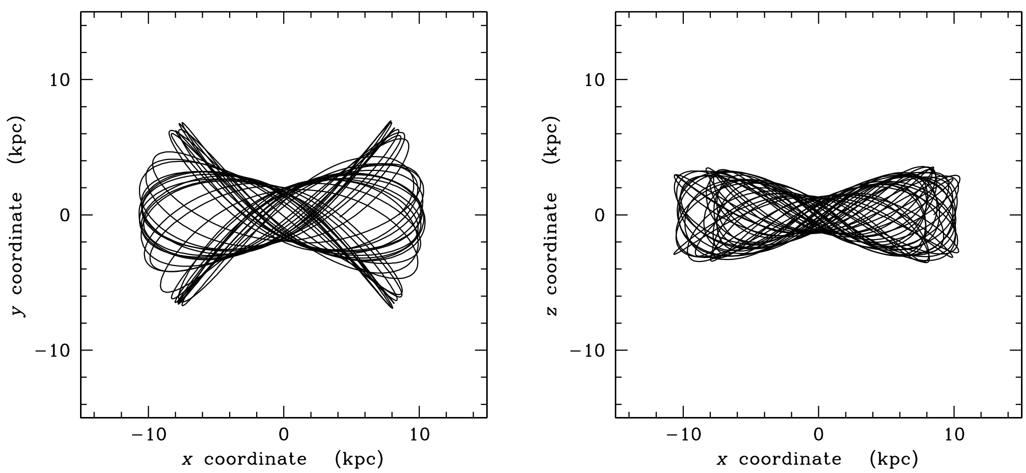
[6 marks]

Question A5

Potential A:



Potential B:



The diagrams above show the orbit of a star in two gravitational potentials, A (upper) and B (lower): each orbit is shown projected in the $x - y$ plane (left) and the $x - z$ plane (right).
What do you conclude about each of the potentials A and B: are they (i) spherical,
(ii) flattened (oblate), or (iii) triaxial? Justify your answer on the basis of the character of each orbit.

[4 marks]

Question A6

Describe the main properties and physical importance of dust in the interstellar medium: refer to its influence on observations, the observational methods by which it is detected, and its influence on star formation.

[5 marks]

Question A7

In Galactic chemical evolution, the symbols X, Y, Z denote the mass fractions of hydrogen, helium and heavy elements respectively. Give typical values of these in the local interstellar medium. What processes respectively produced (a) most of the helium, and (b) most of the heavy elements ?

[4 marks]

Question A8

A star near the Galactic plane has observed apparent magnitudes in the blue and visual bands of $B = 15.48$, $V = 14.58$. Comparison of a spectrum of the star to a library of local star spectra indicates that it has absolute magnitudes $M_B = 3.20$, and $M_V = 2.70$. Given that the reddening ratio for interstellar dust is $A_V/E(B-V) = 3.0$, estimate (i) the reddening of the star, (ii) the V-band extinction in magnitudes, and (iii) the distance to the star in parsecs.

[4 marks]

Question A9

Draw a sketch of the light-curve (flux vs time) for a typical gravitational microlensing event.
The Einstein ring radius for gravitational lensing is given by

$$r_E = \sqrt{\frac{4GM}{c^2}\frac{D_L D_{LS}}{D_S}} \quad ,$$

where M is the lens mass, D_L is the distance from Earth to the lens, D_S is the distance from Earth to the source, and D_{LS} is the distance between lens and source.
Assuming a lens mass $M = 0.1\,M_\odot$, a source in the Large Magellanic Cloud at $D_S = 50\,\mathrm{kpc}$, and a lens at $D_L = 10\,\mathrm{kpc}$, calculate the Einstein radius: express your answer in astronomical units.

[6 marks]

Question A10

Describe the general features of the components of our Milky Way Galaxy, including the disk, bulge/bar, stellar halo and dark halo.

[6 marks]

SECTION B **Answer TWO questions from Section B**

Question B1

An isolated system of N stars is bound by its self-gravity. The ith star has a mass m_i, position vector $\mathbf{x}_i$ and velocity $\dot{\mathbf{x}}_i \equiv d\mathbf{x}_i/dt$ where t is time, and the origin is the centre of mass of the system. The total moment of inertia I of the system is defined as

$$I \equiv \sum_{i=1}^{N} m_i \mathbf{x}_i \cdot \mathbf{x}_i \qquad ,$$

where as usual $\mathbf{x}_i \cdot \mathbf{x}_i$ denotes scalar product.

a) Show that

$$\frac{1}{2}\frac{d^2 I}{dt^2} = \sum_{i=1}^{N} m_i \dot{\mathbf{x}}_i \cdot \dot{\mathbf{x}}_i + \sum_{i=1}^{N} m_i \mathbf{x}_i \cdot \ddot{\mathbf{x}}_i \qquad .$$

[4 marks]

b) Give an expression for T, the total kinetic energy of the system.

[2 marks]

c) Give an expression for the gravitational force on star i due to star j in terms of vectors $\mathbf{x}_i$, $\mathbf{x}_j$. Hence write down an expression for the acceleration $\ddot{\mathbf{x}}_i$ of star i as a sum over $j \neq i$.

[4 marks]

d) Hence, prove that

$$\sum_{i=1}^{N} m_i \mathbf{x}_i \cdot \ddot{\mathbf{x}}_i = -\frac{1}{2} \sum_{i,j,(i \neq j)} \frac{G m_i m_j}{|\mathbf{x}_i - \mathbf{x}_j|} \qquad ;$$

thus deduce the virial theorem,

$$2\langle T \rangle + \langle U \rangle = 0 \quad ,$$

where U is the total gravitational potential energy.

[6 marks]

e) The disk of the Milky Way may be modelled as a double-exponential disk with stellar density given by

$$\rho(R, z) = \rho_0 \, e^{-|z|/h} \, e^{-(R-R_0)/L} \ ,$$

where R, z are Galactocentric cylindrical coordinates, ρ_0 is the local stellar density, h is the scale height and L is the scale length. Observations imply $\rho_0 \simeq 0.1 M_\odot$ pc^{-3}, $h \simeq 250$ pc, $L \simeq 3$ kpc and $R_0 \simeq 8$ kpc. Calculate the total stellar mass of the disk in this model.

[7 marks]

f) Discuss the main observational methods, and uncertainties involved in estimating the disk stellar mass from the model above.

[2 marks]

Question B2

a) The continuity equation for the distribution function f of stars in the six-parameter phase space $(x_1, x_2, x_3, v_1, v_2, v_3)$ of position $\mathbf{x}$ and velocity $\mathbf{v}$ states that

$$\frac{\partial f}{\partial t} + \sum_{j=1}^{3} \left(\frac{\partial}{\partial x_j} \left(f \frac{dx_j}{dt} \right) + \frac{\partial}{\partial v_j} \left(f \frac{dv_j}{dt} \right) \right) = 0 \ ,$$

where t is time.

Derive the collisionless Boltzmann equation

$$\frac{\partial f}{\partial t} + \sum_{j=1}^{3} \left(\frac{dx_j}{dt} \frac{\partial f}{\partial x_j} + \frac{dv_j}{dt} \frac{\partial f}{\partial v_j} \right) = 0$$

from the continuity equation, justifying your assumptions.

[6 marks]

b) Describe the advantages of the Jeans equations relative to the collisionless Boltzmann equation for describing the observed distributions of stars in galaxies.

[3 marks]

c) Derive the first of the Jeans equations,

$$\frac{\partial n}{\partial t} + \sum_{i=1}^{3} \frac{\partial}{\partial x_i} (n \langle v_i \rangle) = 0 \, ,$$

from the collisionless Boltzmann equation, where n is the number density of stars and $\langle v_i \rangle$ is the mean value of the v_i velocity component at a point. (Explain your working and assumptions).

[8 marks]

d) Assume that the Galactic halo is spherical, that it has no net rotation, that its velocity dispersion is isotropic and constant, that it has a potential $\Phi(r) = v_0^2 \ln(r/a)$ where v_0, a are constants, and that it has a stellar density profile of the form $n(r) \propto r^{-l}$ where l is a constant. Using the Jeans equation from the Useful Information above, derive an expression for the velocity dispersion σ of halo stars in the solar neighbourhood in terms of v_0 and l.
If $v_0 = 220$ km s^{-1}, calculate σ for a realistic value of l.

[8 marks]

Question B3

a) Define the symbols HI, HII and H_2 referring to hydrogen in the interstellar medium. For each of these species, describe (a) the typical environments in which they are found; (b) the main processes by which they are observed, including the physical emission/absorption processes and the key wavelengths.

[6 marks]

b) Describe the main processes determining the relative abundance of each state of hydrogen above.

[3 marks]

c) A warm atomic gas cloud has a temperature of $5000K$. Explain why it does not emit a significant amount of Balmer line emission, unless it contains hot stars.

[2 marks]

d) In a region of the Galaxy, the total mass of stars is M_{stars}, the total mass of interstellar gas is M_{gas}, and the mass of heavy elements in the interstellar medium is M_{metals}, while the metallicity of the gas is Z. The changes in these quantities in a small time interval are δM_{stars}, δM_{gas}, δM_{metals} and δZ respectively.
For the Simple Model of galactic chemical enrichment, derive the expression

$$\delta Z \ = \ \frac{\delta M_{\text{metals}}}{M_{\text{gas}}} \ - \ Z \, \frac{\delta M_{\text{gas}}}{M_{\text{gas}}} \ .$$

[4 marks]

e) If δM_{metals} and δM_{stars} above are related by
$\delta M_{\text{metals}} \ = \ - \ Z \, \delta M_{\text{stars}} \ + \ p \, \delta M_{\text{stars}}$, where p is the yield of heavy elements, show that

$$\delta Z \ = \ - \, p \, \frac{\delta M_{gas}}{M_{gas}} \quad .$$

[2 marks]

f) Suppose the Simple Model is modified to allow metal-free gas to accrete into the system (but nothing leaves), so the total mass of stars + gas, M_{tot}, can increase. Show that δZ is now given by

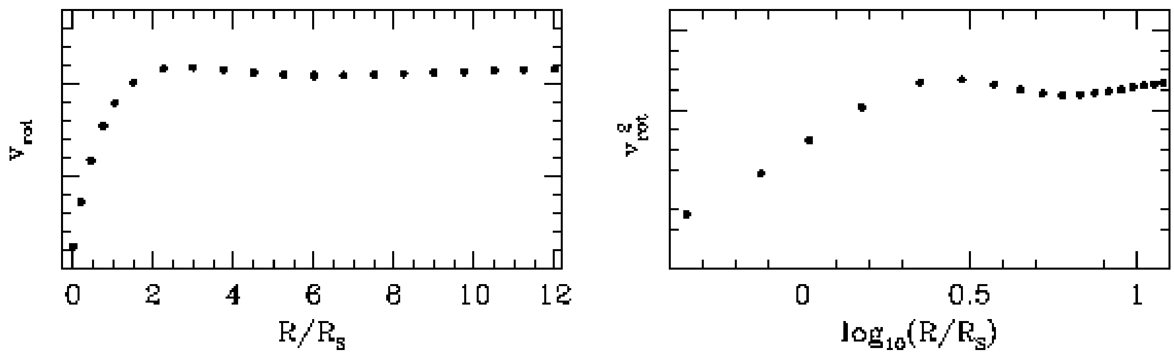
$$\delta Z = \frac{(p - Z)\delta M_{tot} - p \, \delta M_{gas}}{M_{gas}} \ .$$

In the simple case where the gas accretion rate equals the rate at which mass is locked up into stars and stellar remnants, show that Z asymptotes to p. Explain physically why this is expected.

[8 marks]

Question B4

a) The graphs below show the observed rotation curve of one spiral galaxy. The left-hand graph plots the circular velocity v_{rot} against R/R_S, where R is the radial distance from the centre and R_S is the exponential scale length of the galaxy's disc. The right-hand graph plots v_{rot}^2 against $\log_{10}(R/R_S)$.



Comment on the main features of the rotation curve, and interpret this in terms of components of the galaxy (e.g. a bulge, a disc and/or a dark matter halo ?) Explain your reasoning.

[4 marks]

b) Which observational method was probably used to measure the rotation curve in the Figure above ? Explain your reasoning.

[3 marks]

c) In gravitational lensing, the physical Einstein ring radius is given by

$$r_E = \sqrt{\frac{4GM}{c^2}\frac{D_{LS}\,D_L}{D_S}} \quad ,$$

where M is the lens mass, D_L is the distance from Earth to the lens, D_S is the distance from Earth to the source, and D_{LS} is the distance between lens and source.

The optical depth τ to microlensing is defined as the mean number of lenses within 1 r_E of the line of sight to a background source star.
Show that the optical depth τ through a distribution of microlenses of mass M along a line of sight to a given source is given by

$$\tau \;=\; \frac{4\pi G}{c^2 D_S}\int_0^{D_S} D_L\,D_{LS}\,\rho(D_L)\,\mathrm{d}D_L \qquad ,$$

where $\rho(D_L)$ is the mean mass density of lenses at distance D_L.

[9 marks]

d) A survey attempts to detect microlensing events from MACHOs by observing stars in a satellite galaxy at a distance R_f from the Galactic Centre, in the direction of the Galactic anticentre.
Assuming that the dark matter halo of our Galaxy can be represented by an isothermal sphere of compact objects with a density distribution

$$\rho(r) \;=\; \frac{\sigma^2}{2\pi G r^2} \;,$$

where r is the radial distance from the Galactic centre and σ is the velocity dispersion of the compact objects (a constant), show that the optical depth for microlensing the star field is

$$\tau \;=\; \frac{2\sigma^2}{c^2} \left[\left(\frac{R_f + R_0}{R_f - R_0} \right) \ln \left(\frac{R_f}{R_0} \right) \;-\; 2 \right] ,$$

where R_0 is the distance of the Sun from the Galactic centre.

(You may assume the standard integrals

$$\int \frac{x}{(a+x)^2} \, \mathrm{d}x \;=\; \ln \left| a+x \right| \;+\; \frac{a}{a+x} \;+\; \text{constant}$$

and $$\int \frac{x^2}{(a+x)^2} \, \mathrm{d}x \;=\; x \;-\; 2a \ln \left| a+x \right| \;-\; \frac{a^2}{a+x} \;+\; \text{constant} \qquad).$$

[9 marks]

End of Paper