Learning to Learn

Franco Vivaldi
Queen Mary, University of London

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- On average, it takes **two years** to develop such skills.
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- Then you will be an **independent learner**.
- A final year project will give you the opportunity to express these skills in your own unique way.

How would you characterise the derivative of a function?

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gradient

How would you characterise the derivative of a function?

☐ gradient

☐ tangent

How would you characterise the derivative of a function?

☐ gradient

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☐ other

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☐ gradient ☐ tangent ☐ other

Can you compute the derivative of $5x\text{-}2x^3$?

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Can you compute the derivative of $5x\text{-}2x^3$?

☐ yes

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☐ yes ☐ no

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☐ yes ☐ no

Why?

- I hate proofs/proofs are for geeks

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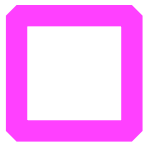

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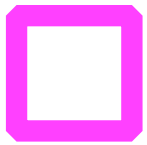

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RIGHT ANSWER:

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☐ yes ☐ no

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RIGHT ANSWER: ✻ **I don't know what is a derivative, and I don't know that I don't know.**

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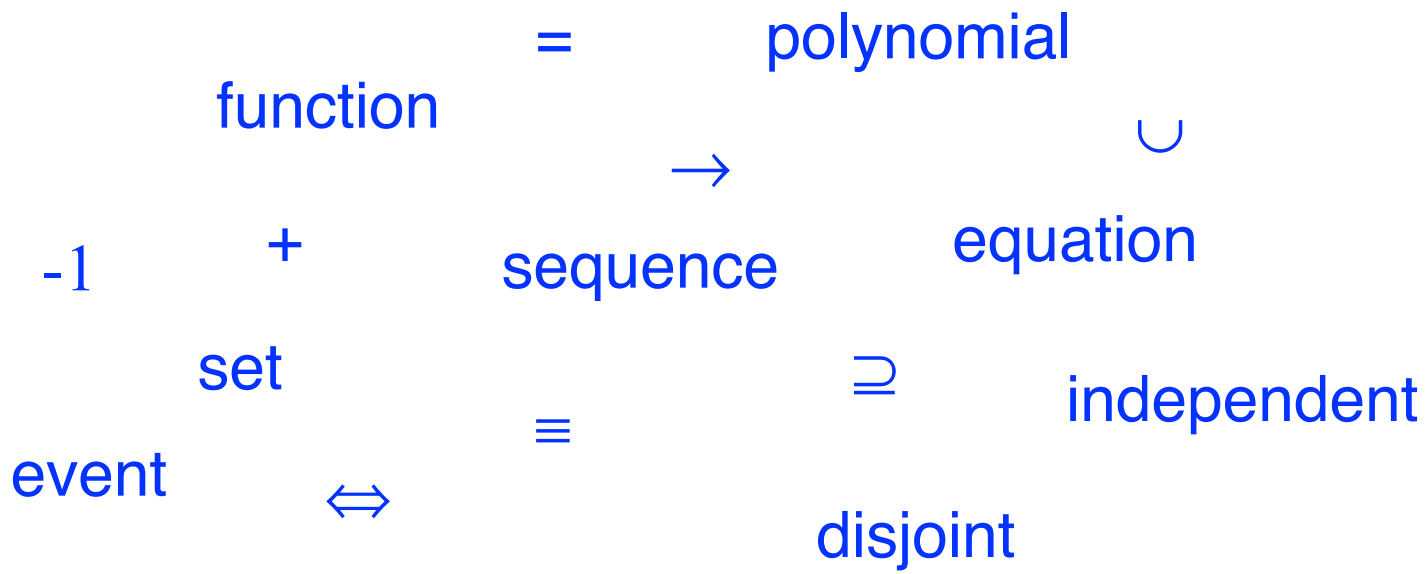

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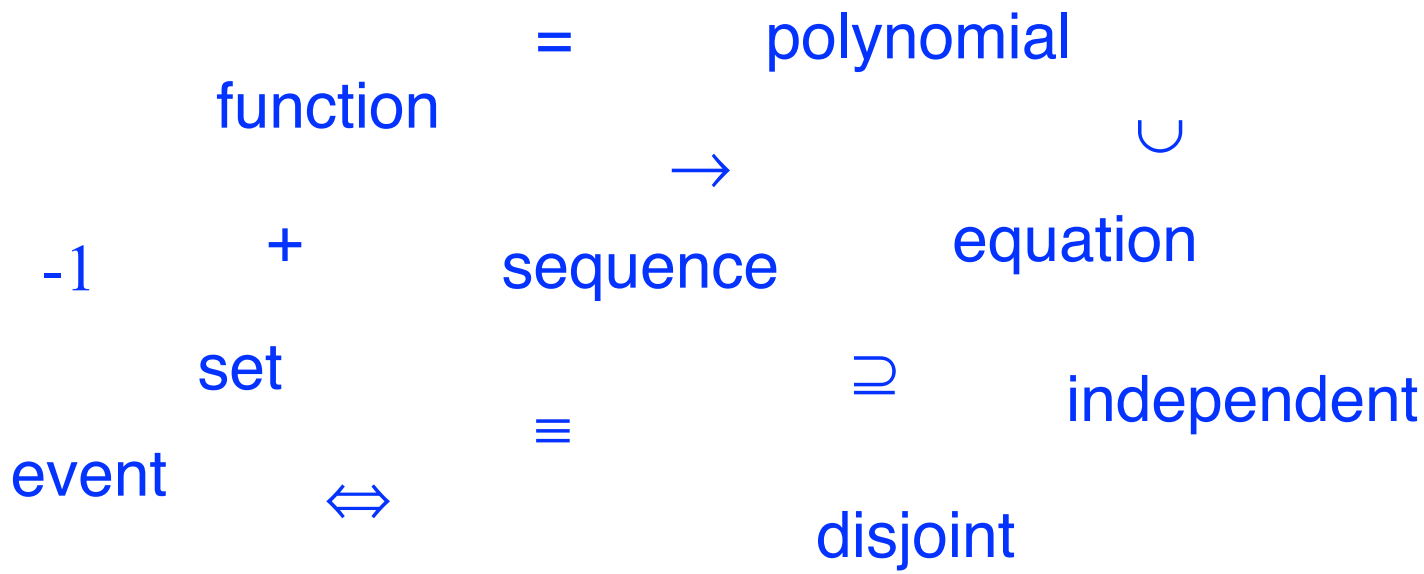

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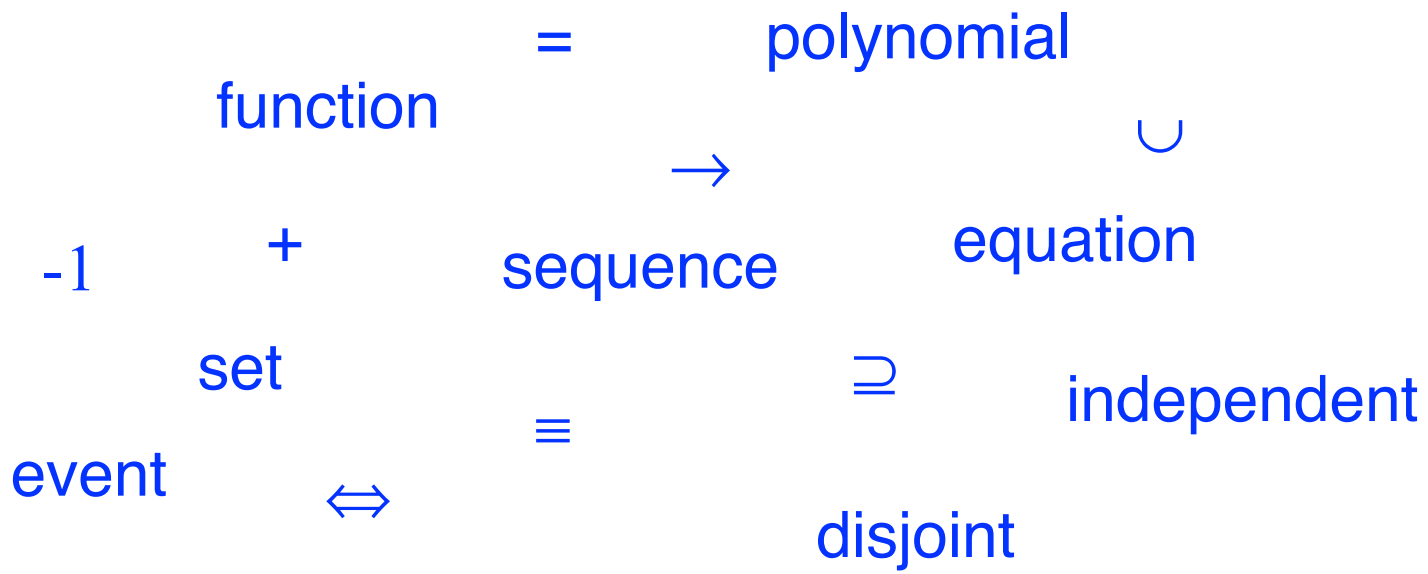

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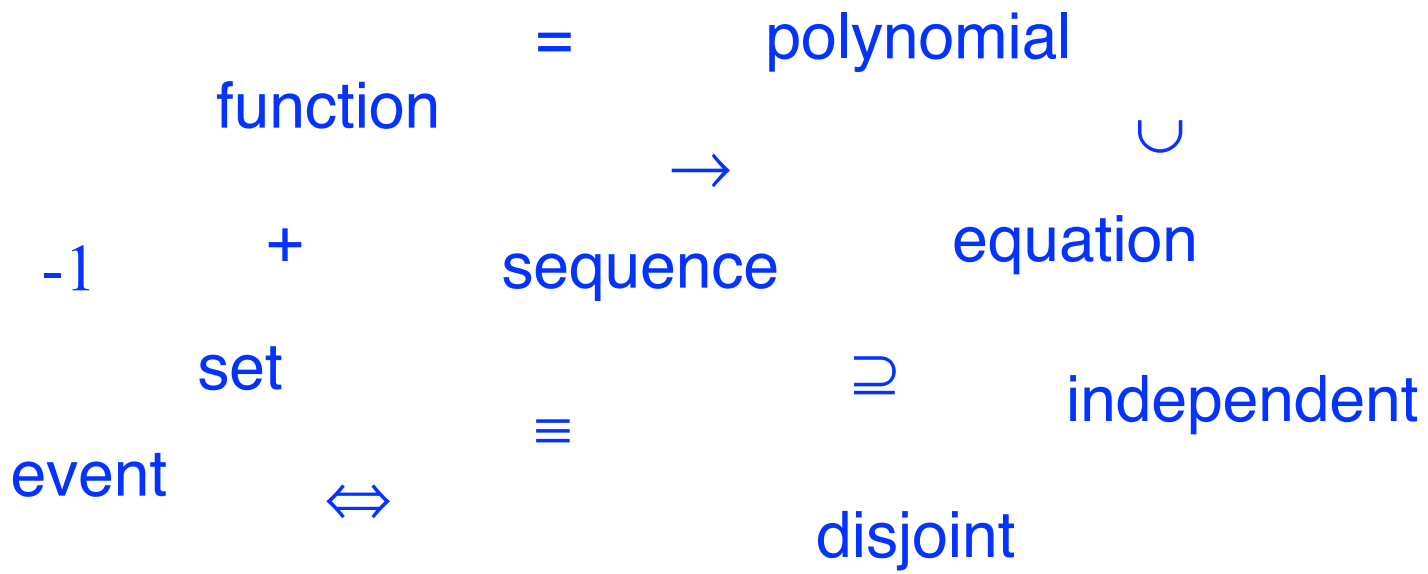

- The language of higher mathematics is beautiful and powerful, but unforgiving.

- The **definitions** are the entries in the mathematical dictionary: they give exact meaning to words and symbols.

- Being able to engage with, analyse, understand, and use definitions is the key to learning higher mathematics.

Mathematical processes: **doing** things

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1. Compute the value of the following expression:

$$\left\{\left(-\frac{2}{3}\right)^2+\left[\left(\frac{1}{5}-\frac{2}{25}\right)\div\left(-\frac{5}{10}+\frac{4}{5}\right)^2-2\right]^3\right\}\times\left(\frac{5}{4}+\frac{5}{8}\right)\div\left(-\frac{5}{3}\right)^2$$

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Computers are better than **humans** in handling processes

Learning how to **do** things

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Then some exercises, similar to the examples

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A worthwhile thing cannot be learned by watching others do it.

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- What is $f(x+h)$ if $f(x)=x^3$?
- Can I solve a **simpler problem**, e.g., $f(x)=x^2$, or $f(x)=x$?

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Learning is when your head hurts in the Library.

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What happens next?

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Robot:

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Robot:

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- **What the f*** does it mean?**

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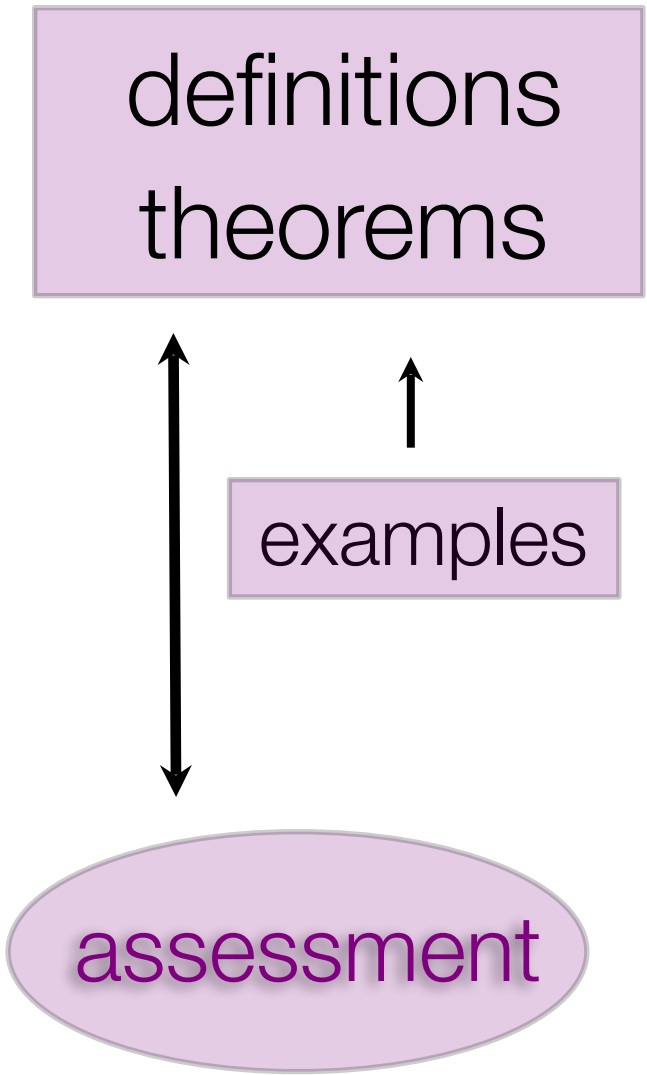

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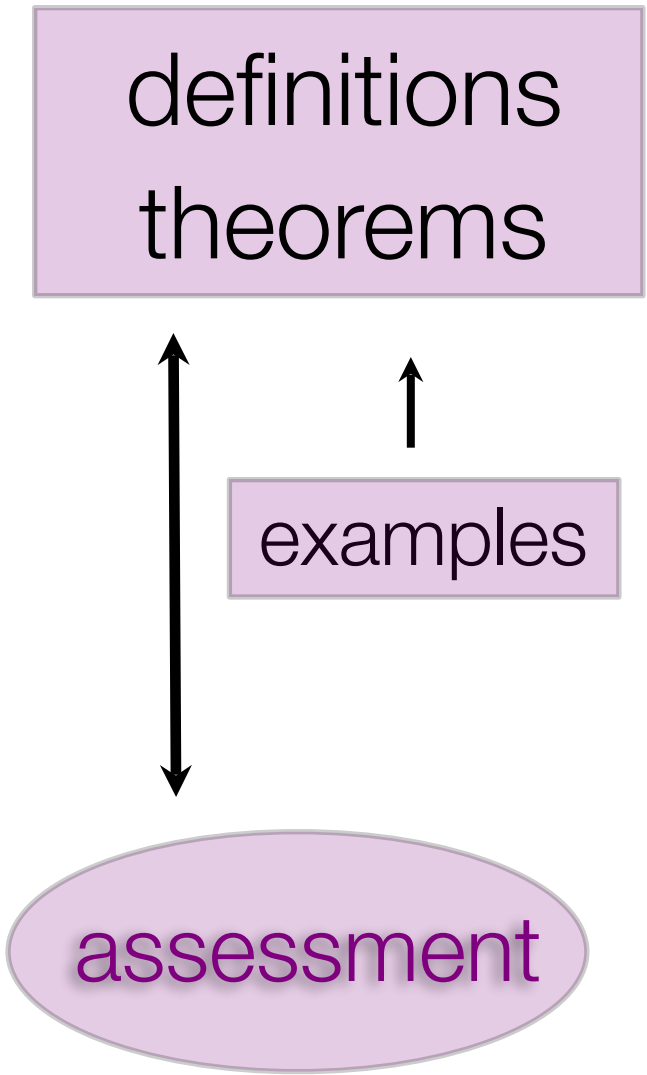

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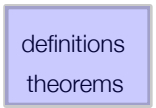

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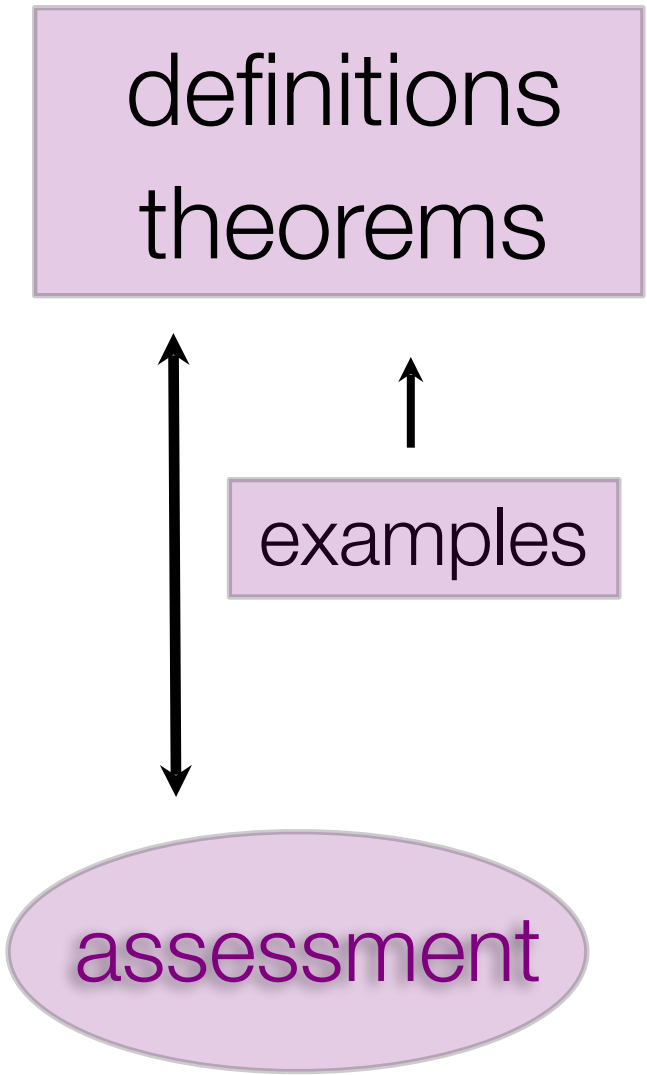

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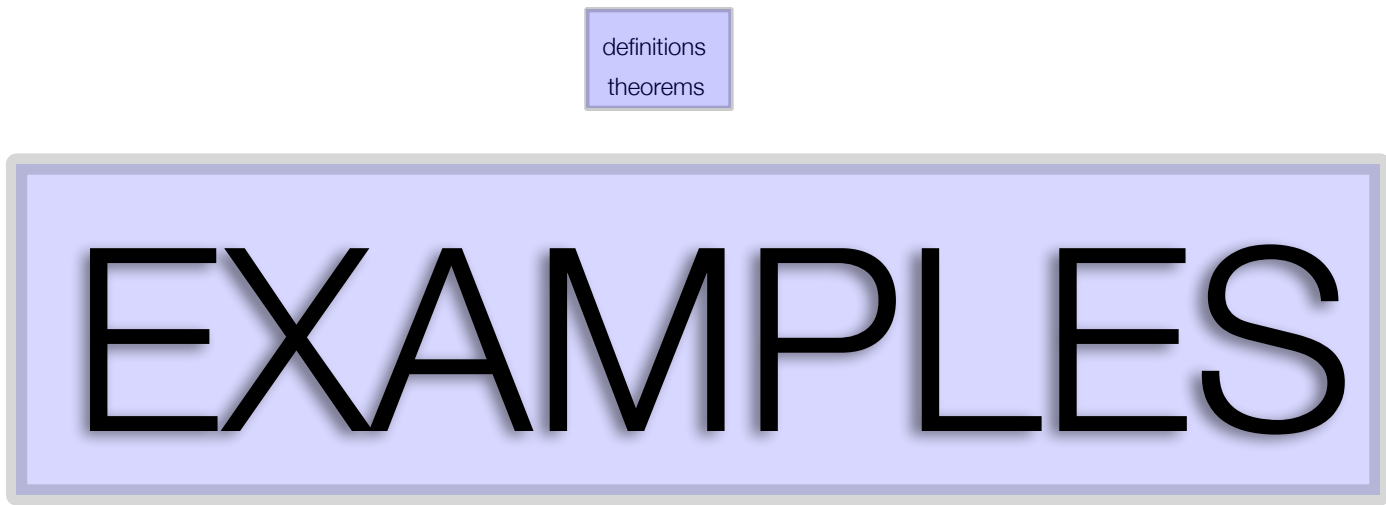

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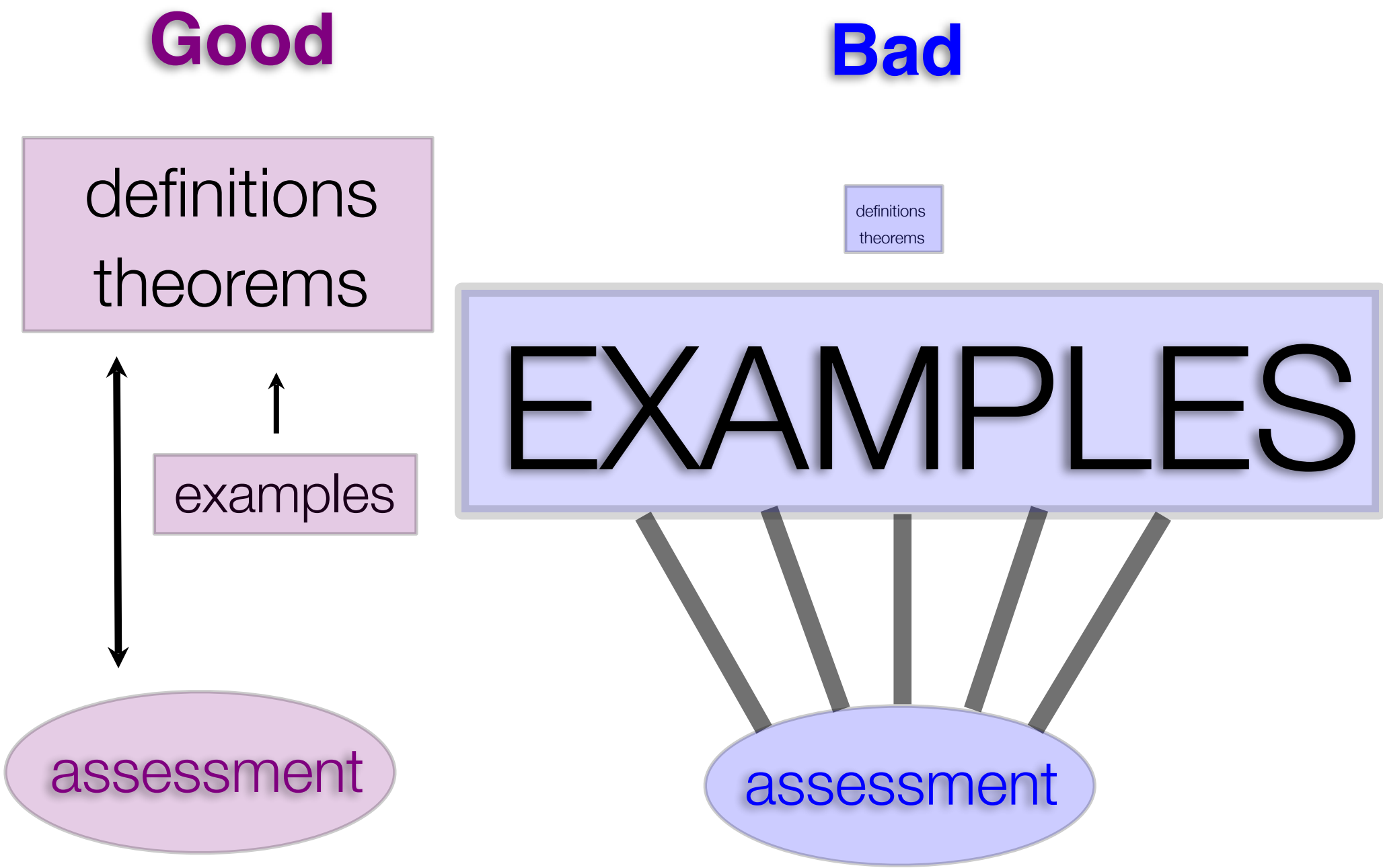

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- The students who did well estimated their mark accurately.

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- The students sat a test, and before being given the result, they were asked to estimate their mark.

The findings

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- The students who did poorly, systematically overestimated their mark, often substantially.

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BIG TIP:

If you don't know the answer to a question, **do not guess**.

The Iceberg Illusion

Success is an iceberg

WHAT PEOPLE SEE

SUCCESS!

WHAT PEOPLE DON'T SEE

Persistence

Dedication

Failure

Hard work

Sacrifice

Discipline

Disappointment

@sylviaduckworth

Thank you for your attention