Chaos & Fractals

Exercises 2

Exercise 1. Order the integers from 1 to 50 inclusive using Sharkovskii's ordering.

Exercise 2. For the map $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = (x-1)(1-3x^2/2)$, determine the orbit of the point 0.

Exercise 3. Use Sharkovskii's Theorem to show that the map $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = (x-1)(1-3x^2/2)$ has a point of least period n for every $n \in \mathbb{N}$.

Exercise 4. Give an example of a continuous map $f : \mathbb{R} \to \mathbb{R}$ which has one fixed point, and no other periodic points.

Exercise 5. Give an example of a continuous map $f : \mathbb{R} \to \mathbb{R}$ which has three fixed points, and no other periodic points.

Exercise 6. Give an example of a continuous map $f : \mathbb{R} \to \mathbb{R}$ which has one fixed point, one orbit of least period two, and no other periodic points.

Exercise 7. Give an example of a map $f : \mathbb{R} \to \mathbb{R}$ which has one orbit of least period three, and no other periodic points.

Exercise 8. For the following values of μ, describe the behaviour of the orbit of the point x_0 under the logistic map $f_\mu(x) = \mu x(1-x)$.

(a) $\mu = 4/5 = 0.8$, $x_0 = 3/5 = 0.6$,

(b) $\mu = 7/5 = 1.4$, $x_0 = 1/2 = 0.5$,

(c) $\mu = 33/10 = 3.3$, $x_0 = 13/20 = 0.65$,

(d) $\mu = 4$, $x_0 = 13/20 = 0.65$,

(e) $\mu = 4$, $x_0 = 33/50 = 0.66$.

Exercise 9. Let $f(x) = 1 - (13/10)x^2 = 1 - 1.3x^2$.

Use a computer to determine numerically the first 50 points in the f-orbit of the point 0, and the first 50 points in the f-orbit of the point $1/3$.

What is the period of the attracting orbit of f?

Exercise 10. Let $f(x) = 1 - \bar{\lambda}x^2$ where the value

$$\bar{\lambda} = \frac{1}{3}\left(2 + \left(25/2 - 3\sqrt{69}/2\right)^{1/3} + \left(25/2 + 3\sqrt{69}/2\right)^{1/3}\right) \approx 1.75487766624669276$$

is the only real root of the polynomial $1 - \lambda + 2\lambda^2 - \lambda^3$. (The value $\bar{\lambda}$ is chosen so that 0 is a period-3 point).

Use a computer to determine numerically the first 100 points in the f-orbit $\{f^n(1/3)\}_{n=0}^{\infty}$ of the point $1/3$.

What is the smallest value $n \in \mathbb{N}$ such that $|f^n(1/3)| < 1/100$?

What is the smallest value $n \in \mathbb{N}$ such that $|f^n(1/3)| < 1/1000$?

What is the smallest value $n \in \mathbb{N}$ such that $|f^n(1/3)| < 10^{-6}$?