

Chaos & Fractals

Exercises 1

(A) Suppose the map $f : \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = x^2 - 6x + 10$.

Exercise 1. Draw the graph of the map f , and determine all its fixed points. Determine which of these points are attracting and which of these points are repelling.

Exercise 2. For the map f , determine an eventually fixed point which is not a fixed point.

Exercise 3. Draw a graph of the map f^2 . Determine all the points of least period 2 of f . Determine which of these points are attracting and which of these points are repelling.

(B) Now suppose the map $f : \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = x^2 - 7/4$.

Exercise 4. Draw the graph of the map f , and determine all its fixed points. Determine which of these points are attracting and which of these points are repelling.

Exercise 5. For the map f , determine an eventually fixed point which is not a fixed point.

Exercise 6. Draw a graph of the map f^2 . Determine all the points of least period 2 of f . Determine which of these points are attracting and which of these points are repelling.

(C) Now suppose the map $f : \mathbb{R} \rightarrow \mathbb{R}$ is defined by

$$f(x) = \begin{cases} x + 1/2 & \text{for } x < 0 \\ -2x + 1/2 & \text{for } x \geq 0. \end{cases}$$

Exercise 7. Draw the graph of the map f , and determine all its fixed points. Determine which of these points are attracting and which of these points are repelling.

Exercise 8. For the map f , determine an eventually fixed point which is not a fixed point.

Exercise 9. Draw a graph of the map f^2 . Determine all the points of least period 2 of f . Determine which of these points are attracting and which of these points are repelling.

Exercise 10. For the map f , determine all its points of least period 3.

Exercise 11. For the map f , determine all its points of least period 4.

Exercise 12. Is it the case that f has a point of least period n for every $n \in \mathbb{N}$?

Exercise 13. Can you guess (or even prove) a formula for the number of points of period n for the map f ?