Properties of Expected Value and Variance

STOCK A r_A RETURNS OF A

STOCK B r_B

w_A WEIGHTS

w_B

$$r_p = w_A r_A + w_B r_B$$

$$E(r_p) = w_A E(r_A) + w_B E(r_B)$$

$$E(aX+bY) = a E(X) + b E(Y)$$

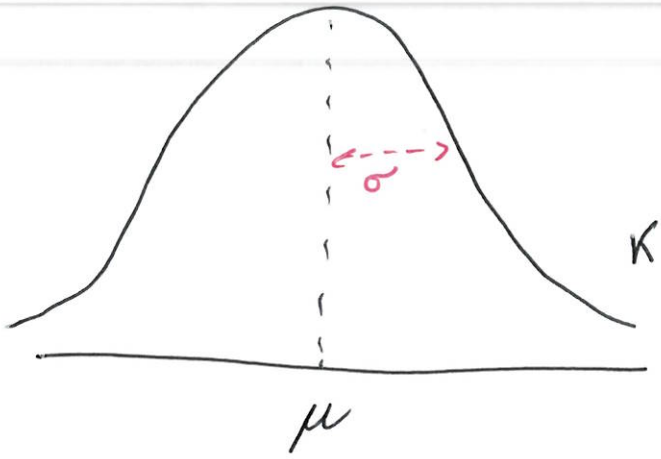
$$Var(r_p) = w_A^2 Var(r_A) + w_B^2 Var(r_B) + 2 w_A w_B Cov(r_A, r_B)$$

$$Var(aX+bY) = a^2 Var(X) + b^2 Var(Y) + 2ab Cov(X,Y)$$

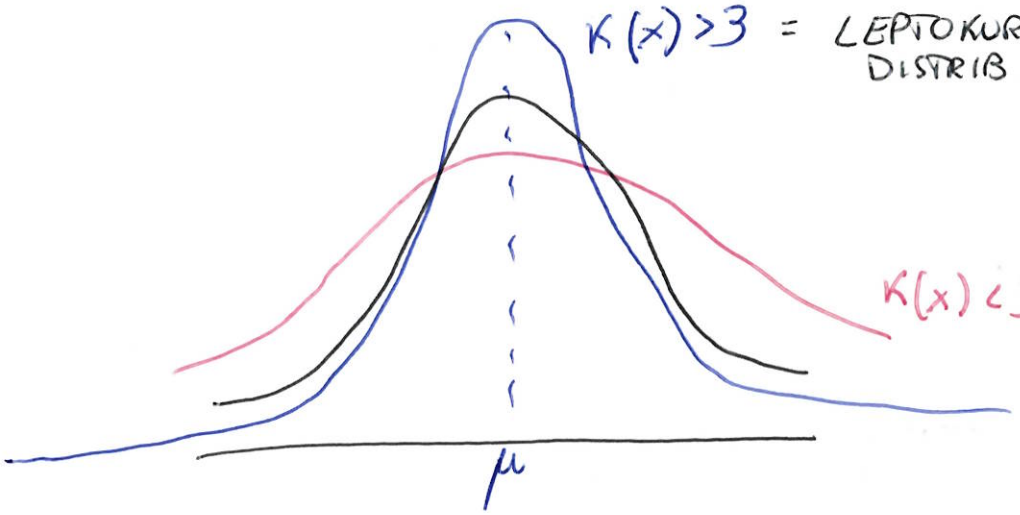
$$r_p = w_a r_A + w_B r_B + w_C r_C$$

Kurtosis

$X \sim N(\mu, \sigma^2)$



$K(x) = 3$ = NORMAL DISTRIB.



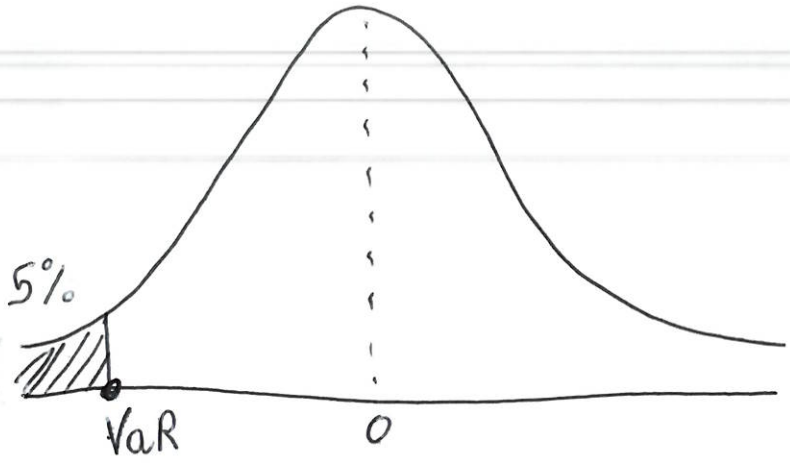
$K(x) > 3$ = LEPTOKURTIC DISTRIB.

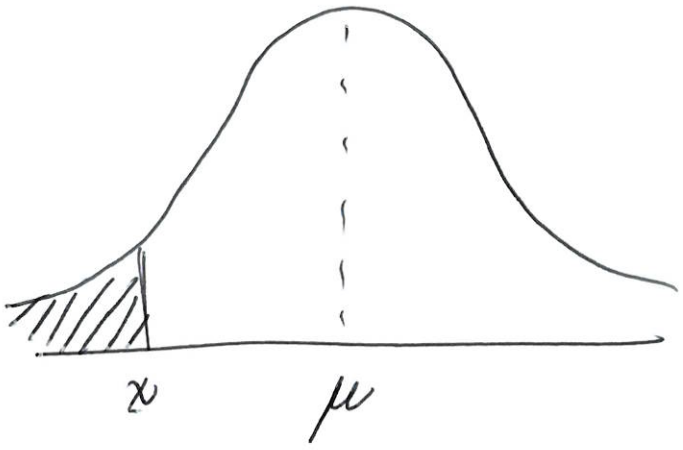
$K(x) < 3$ = PLATYKURTIC DISTRIB.

EXCESS KURTOSIS : $K(x) - 3$

IF $K(x) = 3$ → EXC. KURT. : $= 3 - 3 = 0$

Probability and Normal distribution





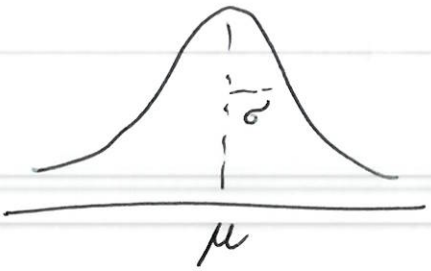
$$P(X < x)$$

$$\int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{(x-\mu)^2}{\sigma^2}\right)}$$

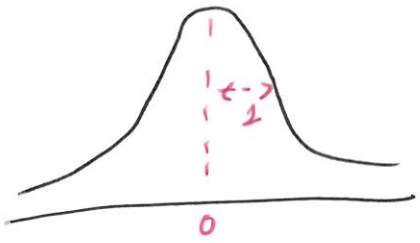
$$P(a \leq X < b) = ?$$

$$\int_{a}^{b}$$

$X \sim N(\mu, \sigma^2)$



$Z \sim N(0, 1)$



$$X \sim N(\mu, \sigma^2)$$

$$Z = \frac{X - \mu}{\sigma} \longrightarrow$$

THIS IS HOW TO TRANSFORM A X VALUE TO A Z-SCORE

Example: How to calculate probabilities?

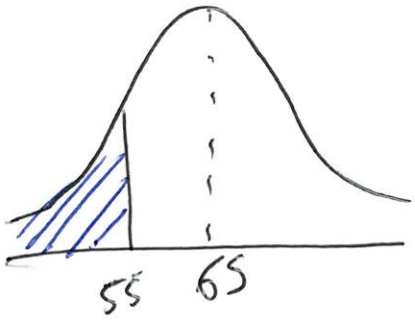
[redacted]

X (MARK)

$\mu_x = 65$

$\sigma_x = 10$

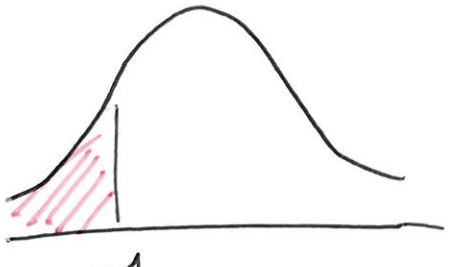
$P(x < 55)$



$$P\left(z < \frac{55 - 65}{10}\right)$$

mean ↓

↑ st. dev)

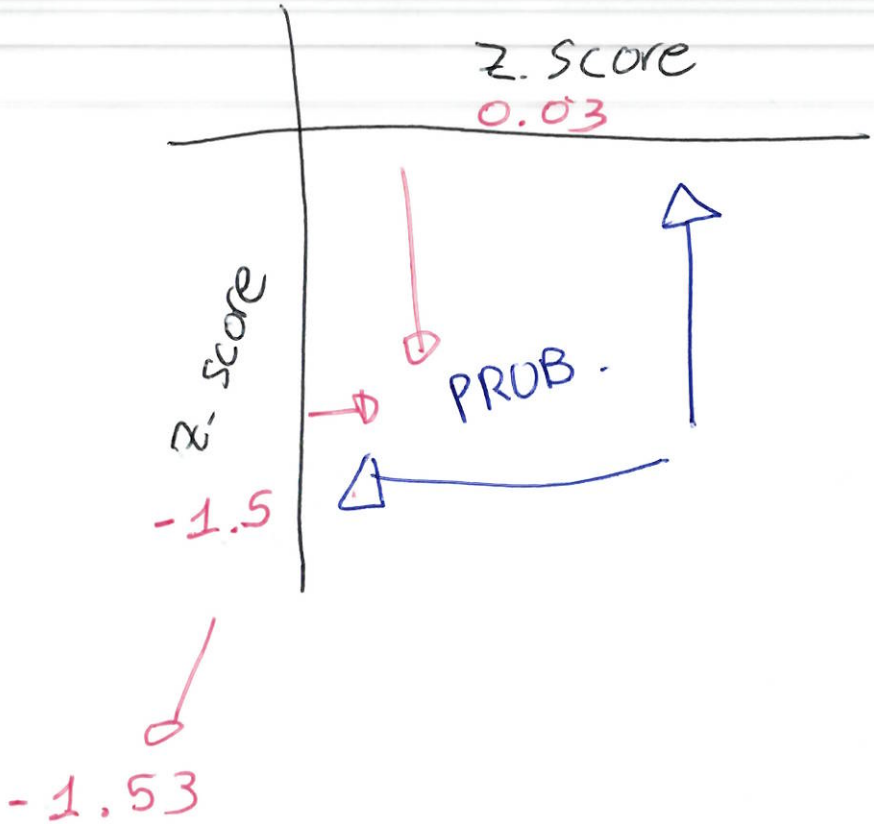


$P(z < -1)$

$= 0.1587$

$= 15.87\%$

HOW TO READ STANDARD NORMAL TABLE



- $P(Z < -1.00) = ?$

$P(Z < z) = 0.1585$

$P(Z < z) = 0.1762$

$z = -0.93$

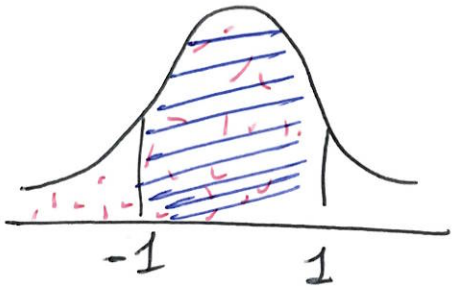
HOW TO CALCULATE PROBABILITIES

$$P(Z < -1)$$

$$P(Z > -1) =$$

$$= 1 - P(Z < -1)$$

$$P(-1 < Z < 1)$$



$$= P(Z < 1) - P(Z < -1)$$

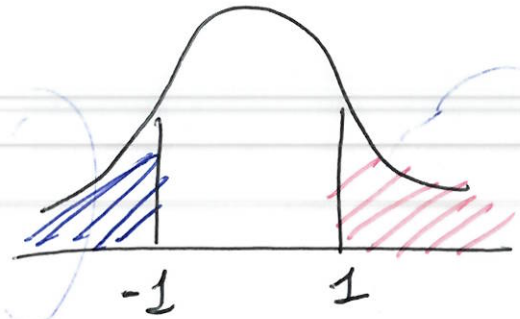
X = MARK IN MATHS

$$P(X > 70)$$

$$= 1 - P(X < 70)$$

$$P(z > 1)$$

$$= 1 - P(z < 1)$$



$$P(z > 1) = P(z < -1)$$

WHAT IS AN ESTIMATOR?

POP. → ALL STUDENTS IN QM

SAMPLE → SUBSET OF POP ($n = 1000$)

AVERAGE MARK (POPULATION) (μ)

μ → PARAMETER — unknown

↓

$\bar{X}$ SAMPLE MEAN

$\bar{X} = \frac{1}{n} \Sigma x_i$ } ESTIMATOR

$\bar{X} = 68$ ESTIMATE

$\mu \rightarrow \hat{\mu} = \bar{X}$

$\sigma \rightarrow \hat{\sigma}$

$\theta \rightarrow \hat{\theta}$