

Maths & Stats Pre-Sessional Tutorial

Topic 6: Regression Analysis

Exercise 1

A large consumer goods company has been studying the effect of advertising on total profits. As part of this study, data on advertising expenditures (X) and total sales (Y) were collected for a 5-month period and are as follows:

(x, y) : (10,100) (15,200) (7,80) (12,120) (14,150).

- a) Plot the data.
- b) Does the plot provide evidence that advertising has a positive effect on sales?
- c) Knowing that $\text{Cov}(x, y) = 140$ and $s_{\text{Cov}}(X, Y) = 140$ and $s_X^2 = 10.3$, compute the regression coefficients $\widehat{\beta}_0$ and $\widehat{\beta}_1$.

Exercise 2

An aircraft company wanted to predict the number of worker-hours necessary to finish the design of a new plane. Relevant explanatory variables were thought to be the plane's top speed, its weight, and the number of parts it had in common with other models built by the company. A sample of 27 of the company's planes was taken, and the following model was estimated:

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + u_i$$

where y_i is the design effort, in millions of worker-hours; x_{1i} is the plane's top speed, in miles per hour; x_{2i} is plane's weight, in tons; x_{3i} is the percentage number of parts in common with other models.

The estimated regression coefficients were as follows:

$$\widehat{\beta}_1 = 0.661$$

$$\widehat{\beta}_2 = 0.065$$

$$\widehat{\beta}_3 = -0.018$$

Provide a definition of estimate and interpret the estimates above.

Exercise 3

Consider the following linear model estimated for an industrial sector:

$$\hat{y} = 10 + 5x_1 + 4x_2 - 2x_3$$

where: y is profit (in thousand \$) for the firm, x_1 is the number of workers, x_2 is the average number of years of education of the firm's workforce, and x_3 is the number of competitors.

- a) Compute $\hat{y}$ when $x_1 = 20$, $x_2 = 11$ and $x_3 = 10$
- b) Compute $\hat{y}$ when $x_1 = 15$, $x_2 = 14$ and $x_3 = 20$

Exercise 4

Consider the following regression of the percentage change in the Dow Jones index in a year on the percentage change in the index over the first 5 trading days of the year:

$$\hat{y} = 12.942 - 2.034x$$

$n = 13$, and the standard error of the slope is 1.378.

- (a) Find a point estimate of the variance of the least squares estimator of the slope of the population regression line.
- (b) Find and interpret a 95% confidence interval for the slope of the population regression line.
- (c) Test at the 10% significance level, against a two-sided alternative, the null hypothesis that the slope of the population regression line is 0