Noncausal Optimal Control of Sea Wave Energy Converters

Dr Guang Li

Outline

- Wave Energy Converters:
  - Point absorber (OPT PB150)
  - Hinged WEC: 1) M4 and 2) Mocean WEC.
  - Onshore WEC
- Control Strategies:
  - Linear Noncausal Optimal Control (LNOC)
  - Model Predictive Control (MPC)

Sea wave energy converter (WEC) control

- **Diverse designs of sea wave energy converters (WECs).**
- **WEC control is challenging!**
    - Lagged behind wind/tidal turbine control.
    - Early WEC control methods are conceptual.
    - No active control methods have been implemented in Sea Trials.
- **My objective in WEC control:** Develop realistically implementable and highly efficient WEC control methods.

Wave Energy Topic 1 – Point Absorber

My early contributions

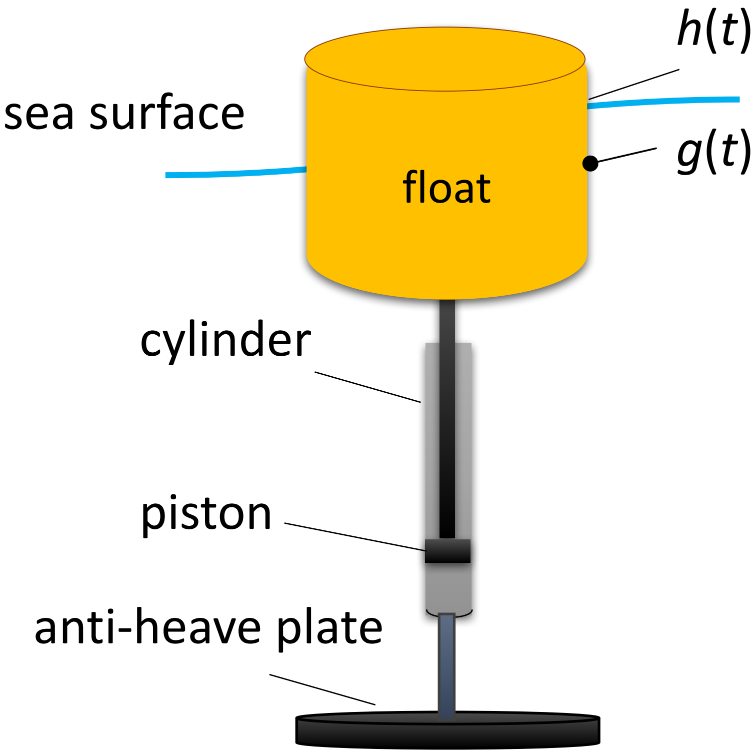
Conventional WEC control methods

Early finding: Maximum energy is extracted when the natural frequency of the WEC is close to the dominant frequency of the incoming waves.

1) Latching control: hold/release the float at proper time according to the incoming wave profile.
2) Declutch control.
3) Impedance matching method: both phase matching and magnitude matching.
4) Passive Damping Control: velocity feedback $u = k_v v$
5) Reactive control: velocity and displacement feedback $u = K_v v + K_x x$.

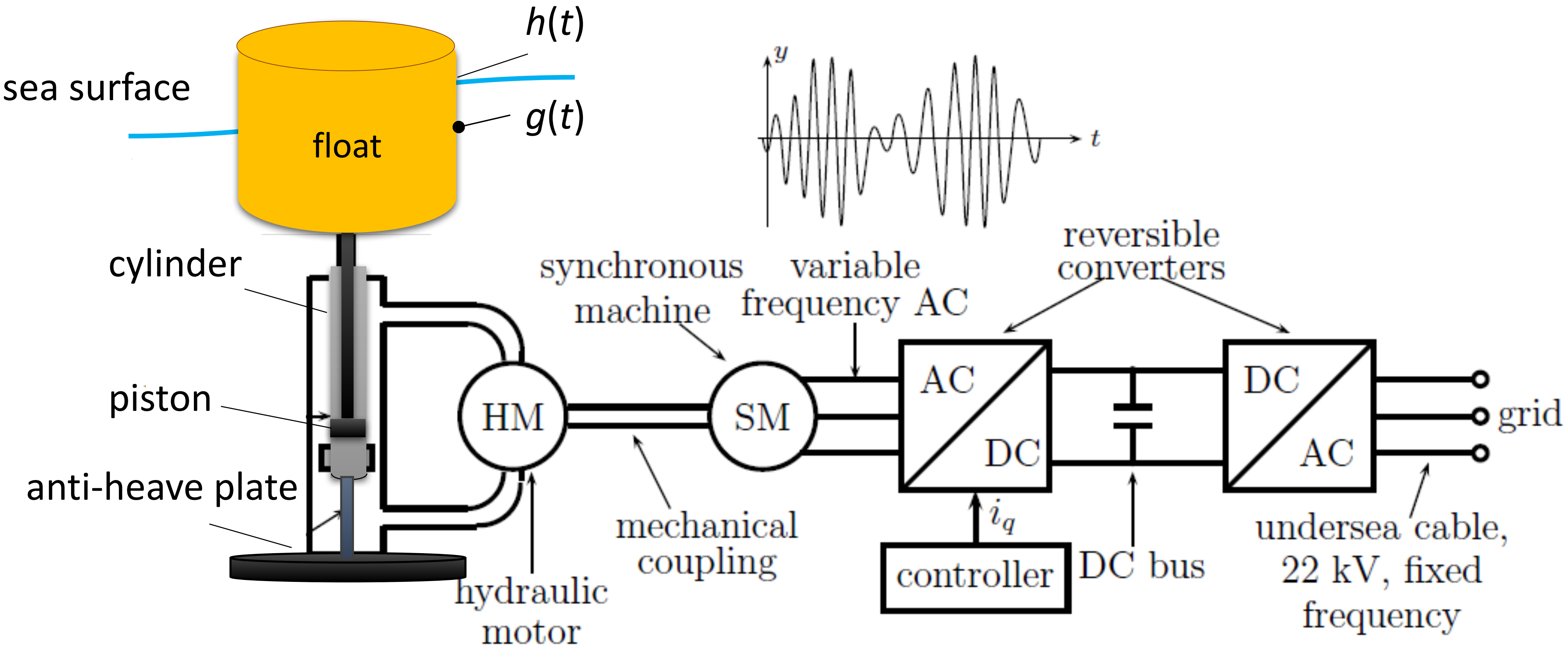
These traditional methods are very inefficient:
highly rely on the prediction of the incoming waves and no feedback is used.

Working principle of point absorber control



Schematic Diagram

Working principle of point absorber control



Schematic Diagram

The point absorber model

The dynamic model can be represented as

$$m\ddot{y} = f_c + f_h + f_r + f_{ext} \qquad (1)$$

where m – float mass;

y – vertical displacement of the float.

f_c – control force;

f_{ext} – wave excitation force;

f_h – buoyancy force, $f_h = -k_h y$;

f_r – radiation force,

$f_r(t) = -\mu_\infty \ddot{y} - \int_{-\infty}^{t} k_r(t-\tau)\dot{y}(\tau)d\tau$ (Cummins equation)

here μ_∞ is the "added mass", representing the force from instantaneous motion, and the convolution represents the forces due to the transient motion.

We can use a LTI model to represent the convolution as

$$\dot{x}_r(t) = A_r x_r(t) + B_r \dot{y}(t)$$

$$y_r(t) = C_r z_r(t) = \int_{-\infty}^{t} k_r(t-\tau)\dot{y}(\tau)d\tau \qquad (2)$$

Optimal Control Formulation

- Control objective: maximize wave energy extraction while maintaining the safe operation.
- Constrained optimal control problem:

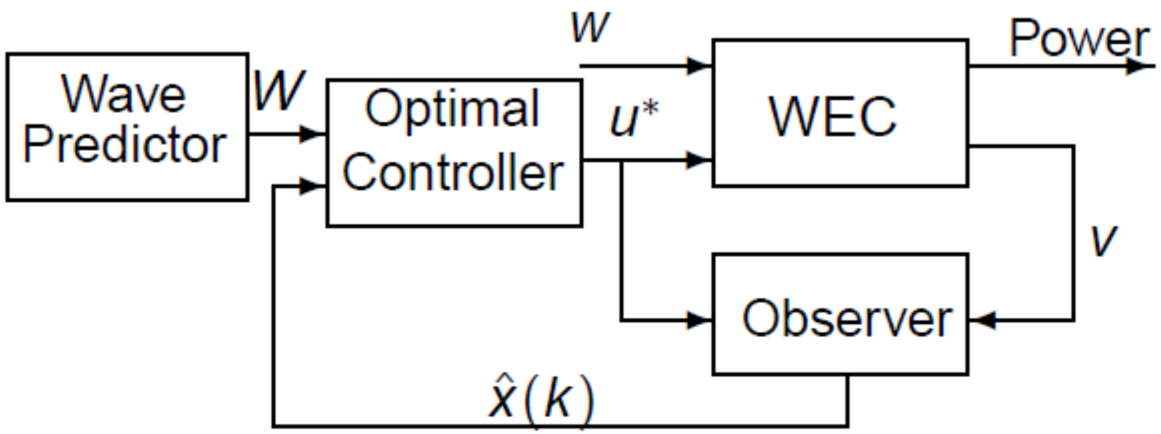
$$\max_{\mathrm{u(t)}} \int_0^T P(t)dt$$

subject to: $\dot{x} = f(x, u, w, t)$ – WEC dynamics

$u_{\min} \leq u(t) \leq u_{\max}$ – Actuator constraint

$z_{\min} \leq z(t) \leq z_{\max}$ – Safety constraint

Implement in a MPC Framework



Resolve a finite horizon constrained optimization problem at each sampling instant

$$\max_{u_0,u_1,\dots,u_N} \sum_{k=0}^{N} v_k u_k$$

Non-convex subject to: $x^+ = f(x,u,w,k)$ – WEC dynamics

$u_{\min} \leq u_k \leq u_{\max}$ – Actuator constraint

$z_{\min} \leq z_k \leq z_{\max}$ – Safety constraint

A Convex Optimization

- Computational speed -- Convex quadratic programme.
- Control signal -- Small and smooth to reduce hardware cost
- Flexible for tuning -- To satisfy large waves.

$$\max_{u_0,u_1,\ldots,u_N} \sum_{k=0}^{N} q v_k u_k + r u_k^2 + s z_k^2$$

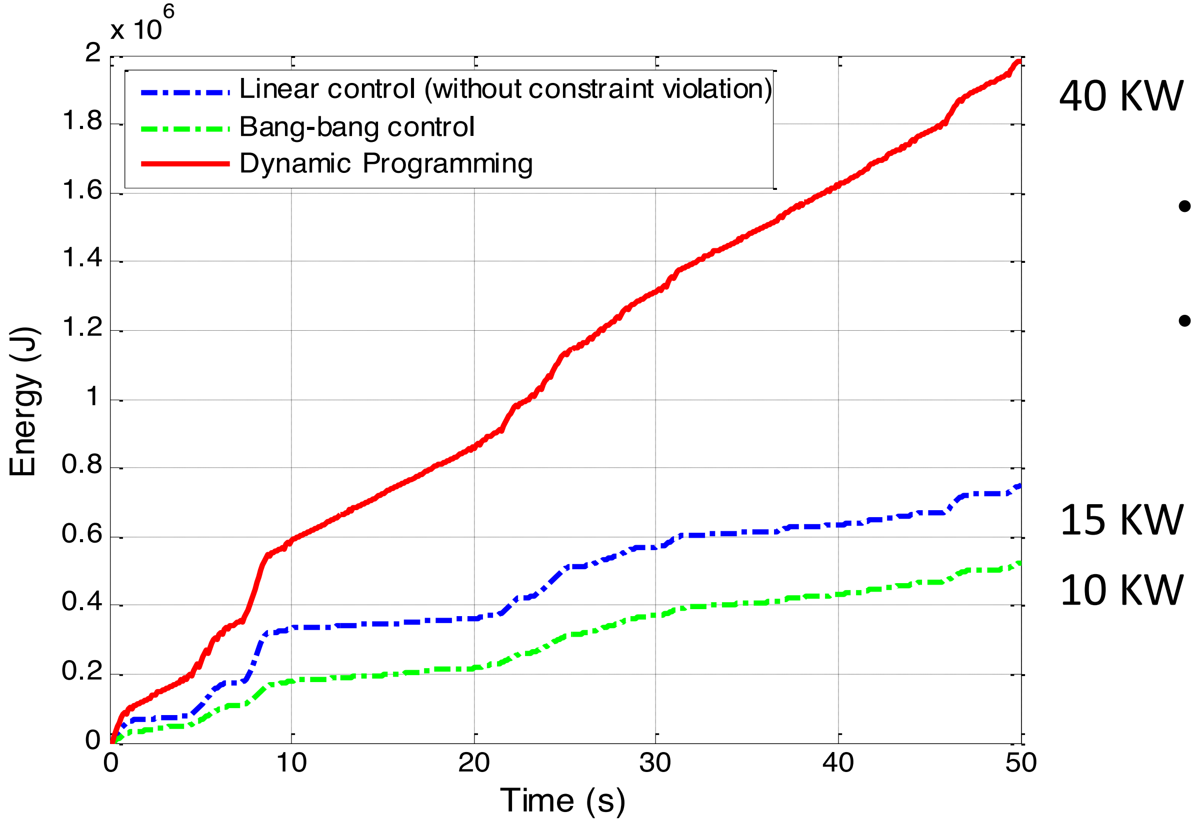
subject to: $x^+ = f(x, u, w, k)$ – WEC dynamics

$u_{\min} \leq u_k \leq u_{\max}$ – Actuator constraint

$z_{\min} \leq z_k \leq z_{\max}$ – Safety constraint

Li G and Belmont MR (2014). **Model predictive control of sea wave energy converters–Part I: A convex approach for the case of a single device.** *Renewable Energy* vol. 69, 453-463.

Our control strategy can double the energy output!



- First realistically feasible WEC control strategy.
- Naturally integrate sea wave prediction.

Figure: Energy comparison when a scaled model is controlled by different methods.

Li G et al. (2012). **Wave energy converter control by wave prediction and dynamic programming.** *Renewable Energy,* vol. 48, 392-403.

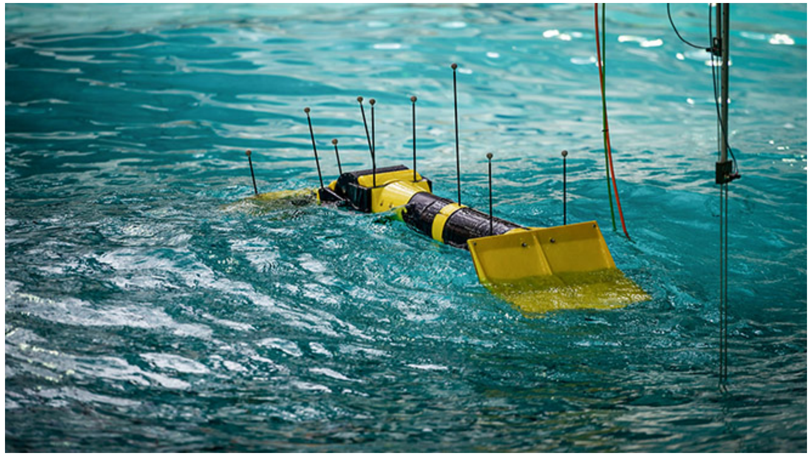
Science News *... from universities, journals, and other research organizations*

Marine Energy Doubled by Predicting Wave Power

June 26, 2012 — The energy generated from our oceans could be doubled using new methods for predicting wave power. Research led by the University of Exeter, published (27 June) in the journal *Renewable Energy*, could pave the way for significant advancements in marine renewable energy, making it a more viable source of power.

New tools for predicting wave power could double the energy from marine renewables. (Credit: © pwollinga / Fotolia)

The study was carried out by a team of mathematicians and engineers from the University of Exeter and Tel Aviv University. They devised a means of accurately predicting the power of the next wave in order to make the technology far more efficient, extracting twice as much energy as is currently



Wave Energy Topic 2 – Hinged Wave Energy Converter

Consortium:

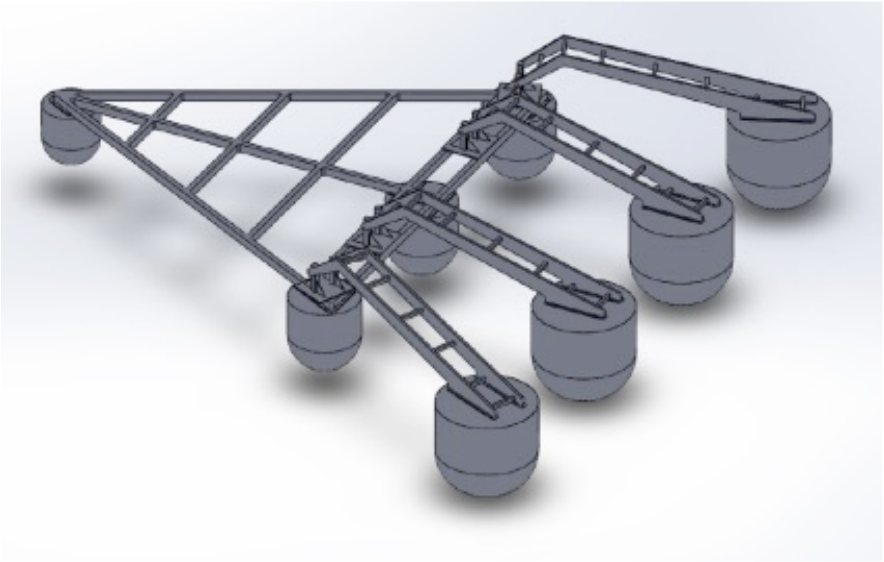
Queen Mary University of London

mocean energy

University of Exeter

Engineering and Physical Sciences Research Council

Wave Energy Topic 3—Control of Multi-Float Multi-Motion and Multi-PTO WEC



- Large capacity multi-float WEC has the potential to produce energy output comparable to a wind turbine.
- Multi-float WEC is challenging to control due to complicated dynamics.

Queen Mary University of London

MANCHESTER 1824 The University of Manchester

UNIVERSITY OF EXETER

UNIVERSITY OF PLYMOUTH

The Control Oriented State-space Model

Hydrodynamic coefficients are calculated using WAMIT.

Define the motion vector as $q := [x_o \; z_o \; \theta_1 \; \theta_2]^\top$

$$
\begin{aligned}
(M + m_\infty)\ddot{q}(t) + f_{rd,q}(t) + Kq(t) &= f_{e,q}(t) + f_{pto,q}(t) \\
\dot{z}_s &= A_s z_s + B_s \dot{q}(t) \\
f_{rd,q}(t) &= C_s z_s + D_s \dot{q}(t)
\end{aligned} \qquad (1)
$$

Radiation damping

$$f_{rd,q}(t) = m_\infty \ddot{q} + \int_{-\infty}^{t} F_{rd}(t-\tau)\dot{q}(\tau)d\tau$$

Convolution by Cummin's method

System identification (order of $16 \times n$)
Model order reduction (order of $16 \times 8 = 128$)

The Control Oriented State-space Model

Define a new state vector as $x := [q,\ \dot{q},\ z_s]^T$

$$\begin{aligned} \dot{x} &= Ax + B_w f_{e,q}(t) + B_u f_{pto,q}(t) \\ z &= Cx \end{aligned} \tag{2}$$

$$A = \begin{bmatrix} 0_{4\times4} & I_{4\times4} & 0_{4\times n} \\ -(M+m_\infty)^{-1}K & -(M+m_\infty)^{-1}D_s & -(M+m_\infty)^{-1}C_s \\ 0_{n\times4} & B_s & A_s \end{bmatrix}$$

$$B_w = \begin{bmatrix} 0_{4\times4} \\ (M+m_\infty)^{-1} \\ 0_{n\times4} \end{bmatrix} \qquad B_u = \begin{bmatrix} 0_{4\times1} \\ (M+m_\infty)^{-1}[0,0,1,-1]^\top \\ 0_{n\times1} \end{bmatrix} \qquad C = \begin{bmatrix} 0_{1\times6} & 1 & -1 & 0_{1\times n} \end{bmatrix}$$

Of order $128 + 8 = 136$

The Control Problem Formulation

Objective： maximizing the WEC energy output.

Following a standard optimal control problem formulation,

$$\min_{u_0,\ldots,u_N} \Sigma_{k=0}^{N} \left\{ z_k u_k + \frac{1}{2} x_k^T Q x_k + \frac{1}{2} r u_k^2 \right\} \qquad (3)$$

s.t. $$x_{k+1} = Ax_k + B_w w_k + B_u u_k$$
$$z_k = Cx_k \qquad (4)$$

- Power output of the WEC

$$P_k = -z_k u_k \qquad z = \dot{\theta}_1 - \dot{\theta}_2$$

where u_k is the PTO torque, z_k is the relative pitch velocity.

"Two different strategies may be attempted to solve these non-realisable (non-causal) optimum-control problems in an approximate manner."
--- Johannes Falnes[2]

[2] J. Falnes, Ocean Waves and Oscillating Systems: Linear Interactions Including Wave-Energy Extraction. Cambridge University Press, 2002.

Linear Noncauasl Optimal Control (LNOC)

The non-causal control law is in the form of

$$u_k = K_x x_k + K_d w_{k,n_p}$$

Feedback Feed-forward

K_x, K_d are time independent.

$$w_{k,n_p} := [w_k, w_{k+1}, \ldots, w_{k+n_p-1}]^\top$$

n_p is the prediction horizon.

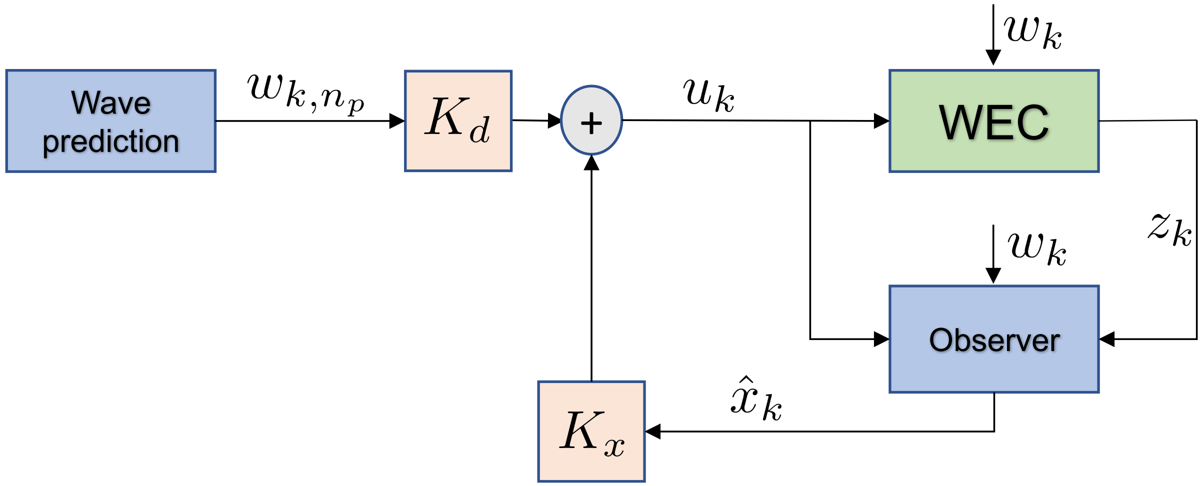


Fig.4 Linear non-causal optimal control framework. [4]

- Wave excitation force prediction can be realized by techniques like DSWP[3].
- In this work prediction is assumed to be perfect.

[3] L. Abusedra and M. Belmont, "Prediction diagrams for deterministic sea wave prediction and the introduction of the data extension prediction method," vol. 58, pp. 59–81, 01 2011.

[4] S. Zhan and G. Li, "Linear noncausal optimal control of wave energy converters," IEEE Transactions on Control System Technology, 2018

Deriving the LNOC control law

Define the optimal cost-to-go function as

$$v(x,k) = \frac{1}{2} x_k^\top V_k x_k + x_k^\top s_k + a_k \tag{5}$$

From Bellman optimality principle

$$v(x,k) = \min_{u_k} \{ L(x_k, u_k) + v(x, k+1) \} \tag{6}$$

The control input at time k:

$$V_N = 0$$
$$s_N = 0$$
$$a_N = 0$$

$$\begin{aligned} u_k &= -(r + B_u^\top V_{k+1} B_u)^{-1} [(B_u^\top V_{k+1} A + C) x_k + B_u^\top V_{k+1} B_w w_k + B_u^\top s_{k+1}] \\ &= K_{x,k} x_k + K_{w,k} w_k + K_{s,k} s_{k+1} \end{aligned} \tag{7}$$

$$V_k = Q + A^\top V_{k+1} A - (C + B_u^\top V_{k+1} A)^\top (r + B_u^\top V_{k+1} B_u)^{-1} (C + B_u^\top V_{k+1} A) \tag{8}$$

$$s_k = (A + B_u K_{x,k})^\top (V_{k+1} B_w w_k + s_{k+1}) \tag{9}$$

Deriving the LNOC control law

If the controller is stable,

$$V = Q + A^\top VA - (C + B_u^\top VA)^\top (r + B_u^\top VB_u)^{-1}(C + B_u^\top VA) \qquad (9)$$

$$u_k = K_x x_k + K_w w_k + K_s s_{k+1} \qquad (10)$$

Rewrite $s_k = \Phi s_{k+1} + \Phi VB_w w_k$ where $\Phi = (A + B_u K_x)^\top$

$$s_{k+1} = \Phi s_{k+2} + \Phi VB_w w_{k+1}$$

$$s_{k+2} = \Phi s_{k+3} + \Phi VB_w w_{k+2}$$

$$s_{k+1} = \Phi^2 s_{k+3} + \Phi^2 VB_w w_{k+2} + \Phi VB_w w_{k+1}$$

Suppose we have prediction horizon up to n_p steps

$$s_{k+1} = \Phi^{n_p-1} s_{n_p} + [0, \Phi VB_w, \dots, \Phi^{n_p-1} VB_w][w_k, w_{k+1}, \dots, w_{k+n_p-1}]^\top \quad (11)$$

Deriving the LNOC control law

Recall that

$$
\begin{aligned}
K_x &= -(r + B_u^\top V B_u)^{-1}(B_u^\top V A + C) \\
K_w &= -(r + B_u^\top V B_u)^{-1} B_u^\top V B_w \\
K_s &= -(r + B_u^\top V B_u)^{-1} B_u^\top
\end{aligned} \quad (7)
$$

$$s_{k+1} = \Phi^{n_p-1} s_{n_p} + [0, \Phi V B_w, \ldots, \Phi^{n_p-1} V B_w][w_k, w_{k+1}, \ldots, w_{k+n_p-1}]^\top \quad (11)$$

$$\Phi = (A + B_u K_x)^\top$$

Define: $\Psi = [V B_w, \Phi V B_w, \ldots, \Phi^{n_p-1} V B_w]$

$$K_d = -(r + B_u^\top V B_u)^{-1} B_u^\top \Psi$$

$$w_{k,n_p} = [w_k, w_{k+1}, \ldots, w_{k+n_p-1}]^\top$$

$$u_k = K_x x_k + K_w w_k + K_s s_{k+1}$$

$$u_k = K_x x_k + K_s(\Phi^{n_p-1} s_{n_p} + \Psi w_{k,n_p})$$

$$u_k = K_x x_k + K_d w_{k,n_p}$$

Numerical Results

$$u_k = d_{pto} z_k$$

vs

$$u_k = K_x x_k + K_d w_{k,n_p}$$

Simulations are run with irregular JONSWAP wave spectrum,

- Significant wave height Hs from 0.04m to 0.07m
- Wave peak period Tp from 0.7s to 2.0s.

Results are compared to those without applying any control, which means the PTO is a well-tuned passive damper.

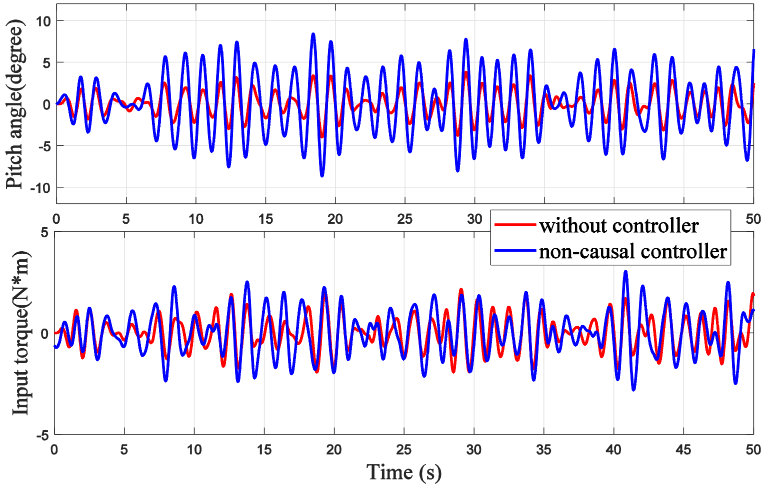


Fig.5 Relative pitch and input torque, Hs=0.04m, Tp=1.8s, prediction horizon 2Tp=3.6s.

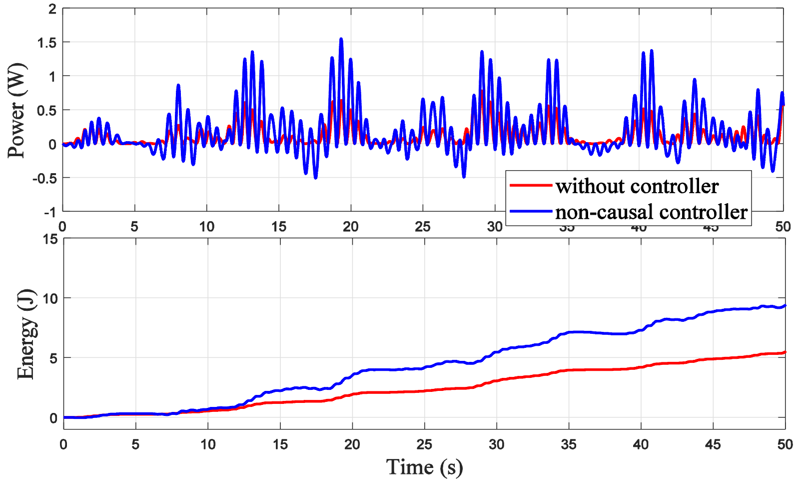


Fig.6 Power and energy, Hs=0.04m, Tp=1.8s, prediction horizon 2Tp=3.6s.

80% energy improvement.

Improvement by 40% to 100% in different sea states for 3-float and 1-PTO M4

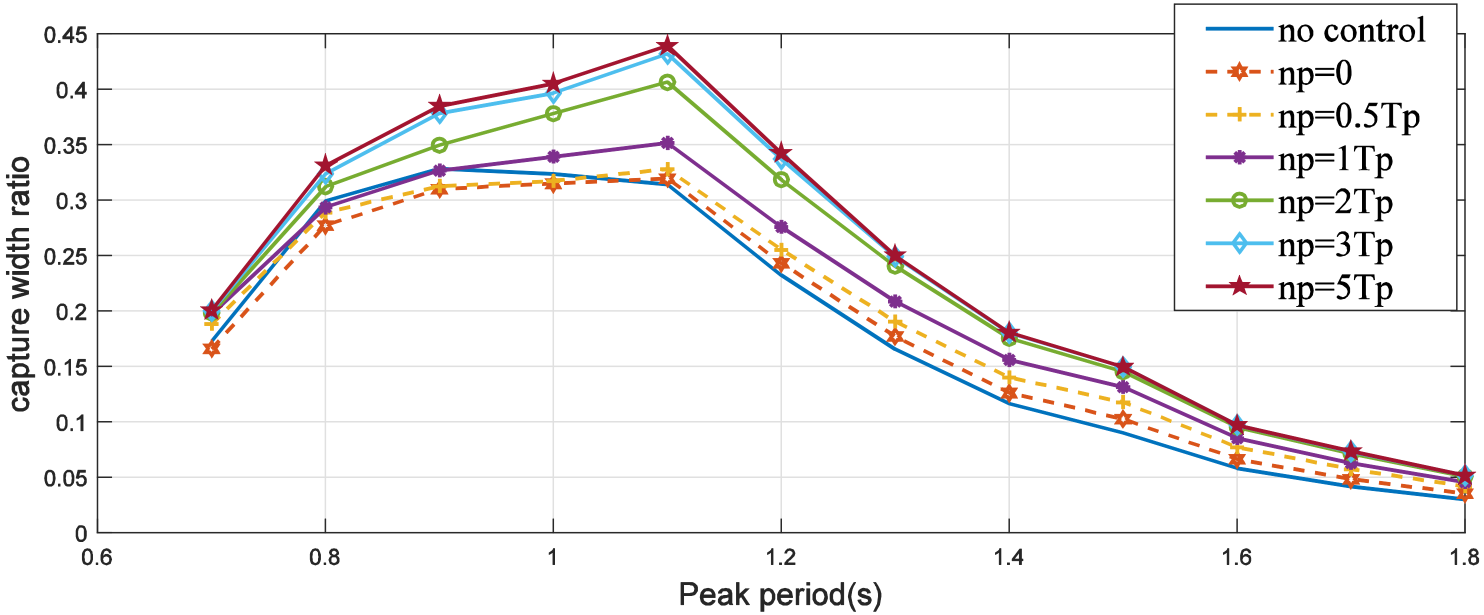


Fig.7 Capture width ratio of different forward wave prediction horizon.

For peak periods 1-1.8s (6-11s ocean waves) Capture width ratio (CWR) increases by 40% to 100%.

Z. Liao, P. Stansby, **G. Li***, Linear non-causal optimal control of an attenuator type wave energy converter M4, *IEEE Transactions on Sustainable Energy*, vol. 11, no. 3, pp. 1278-1286, July 2020.

Z. Liao, P. Stansby and **G. Li***, A generic linear non-causal optimal control framework integrated with wave excitation force prediction for multi-mode wave energy converters with application to M4, *Applied Ocean Research*, vol. 97, 102056, 2020.

Control of multi-float, multi-mode M4 with multiple PTOs – Further progress

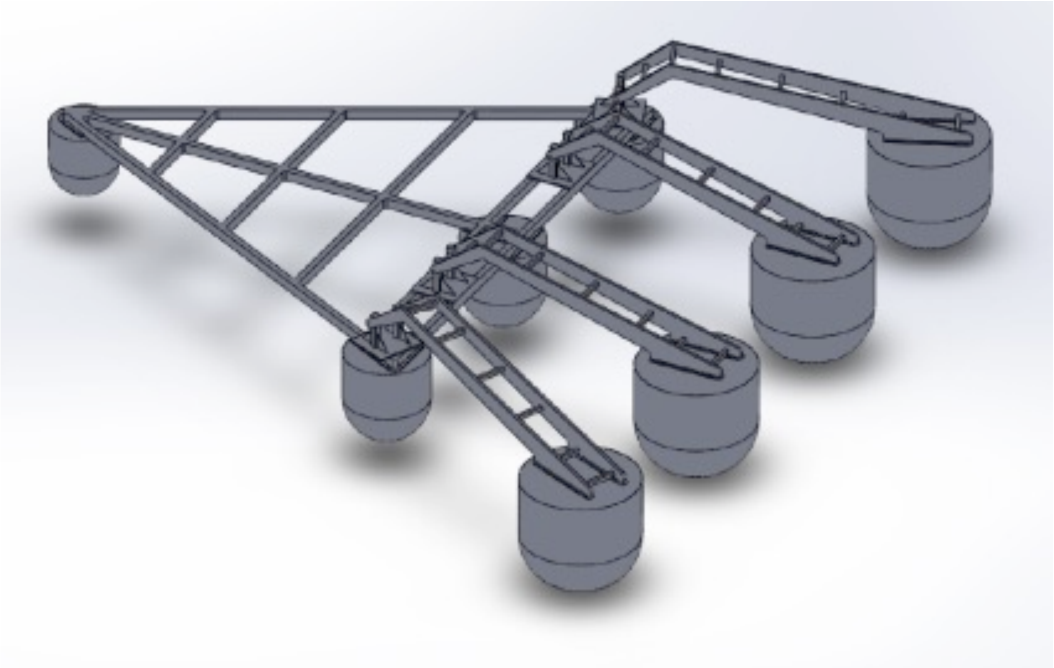


Fig. 1. 3-D view of the 8 floats 4 PTOs, M4 configuration.

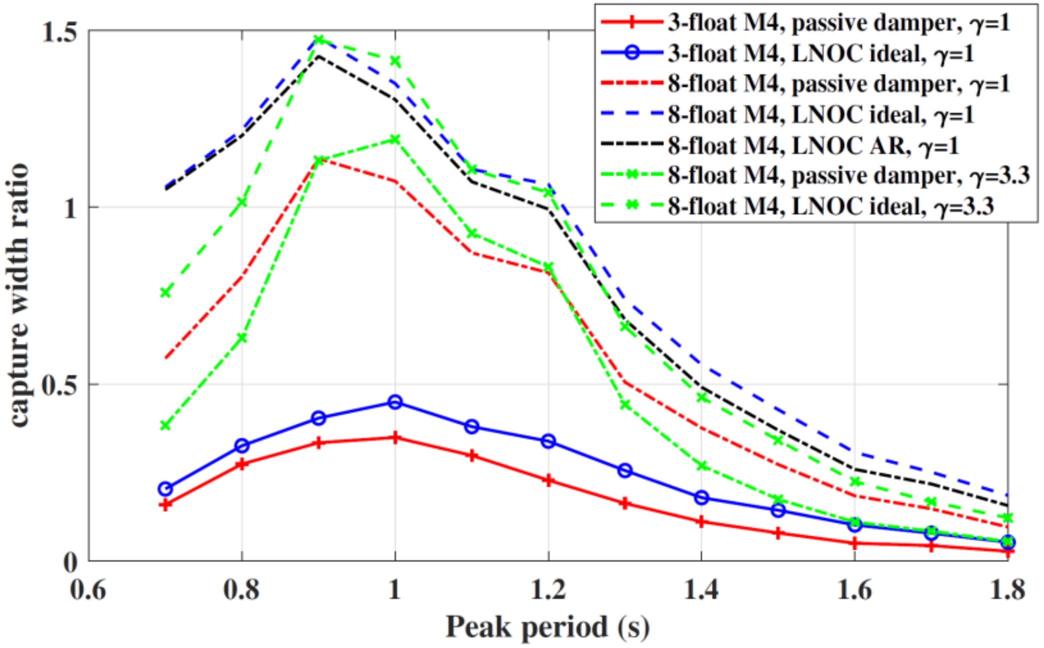


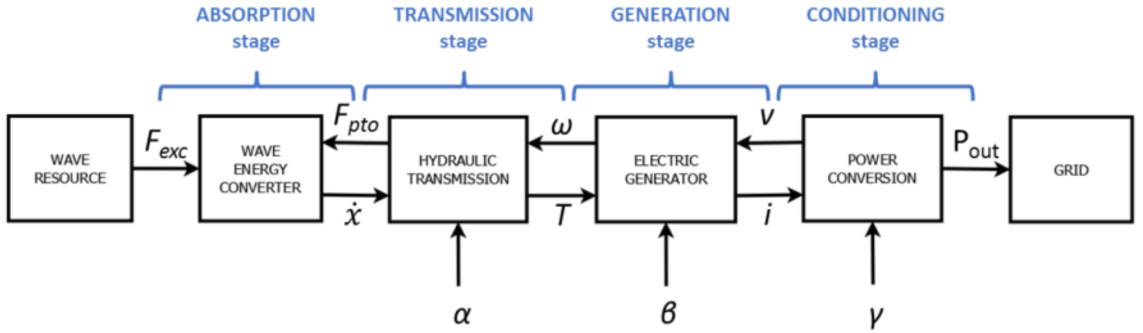
Fig. 4. Capture width ratio comparison: the 3-float M4 versus the 8-float M4.

Z. Liao, P. Stansby, **G. Li***, High-capacity wave energy conversion by multi-floats, multi-PTO, control and prediction: generalised state-space modelling with linear optimal control and arbitrary headings, *IEEE Transactions on Sustainable Energy*.

Whole system-level control system design – To be started soon

EPSRC project (£1M) to be started from September, 2021.
Led by Queen Mary, in collaboration with Manchester and Exeter.

- WEC system consists of several stages:



- WEC performance relies on the efficiency of the whole WEC system.
- We aim to develop control system for the whole system.

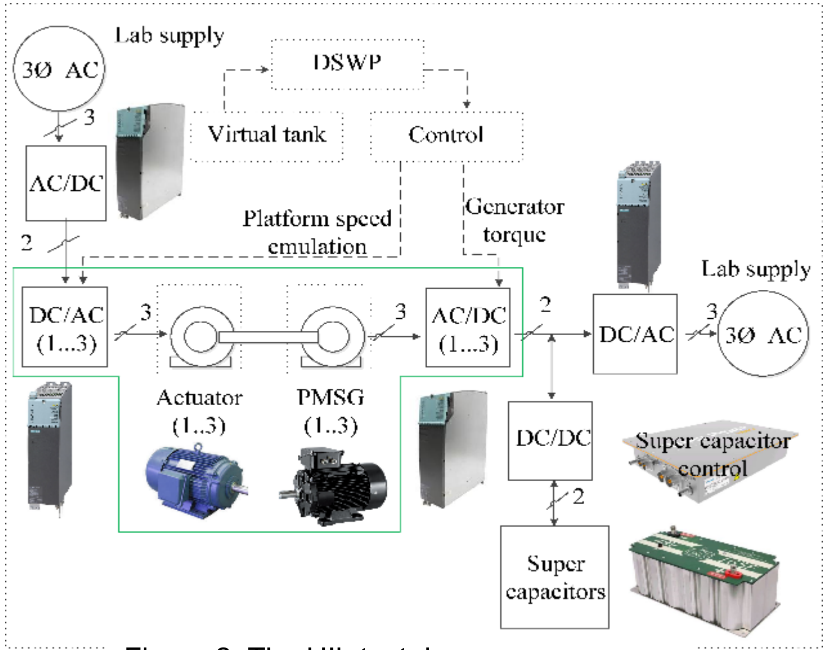


Figure 2: The HIL test rig

A good starting point to get Durham colleagues involved in this research.

The next step for multi-float multi-PTO WEC

Next objectives:

- Tank testing of the scaled physical model.
- Sea trial of the whole system.

Targeting funding sources: Innovate UK, Wave Energy Scotland, EU and overseas funding.

Our planned testing activities:

- Tank testing: 1 to 40^{th} scale, Plymouth COAST lab (this November)
- Sea trial: Validation of quarter-scale 3-float, 1PTO M4 in South China Sea (Tsinghua University).
- Sea trial: Validation of quarter-scale 8-float, 4-PTO M4 control in Australia (University of Western Australia).

The 1st tank testing results at FloWave show the energy output can be doubled

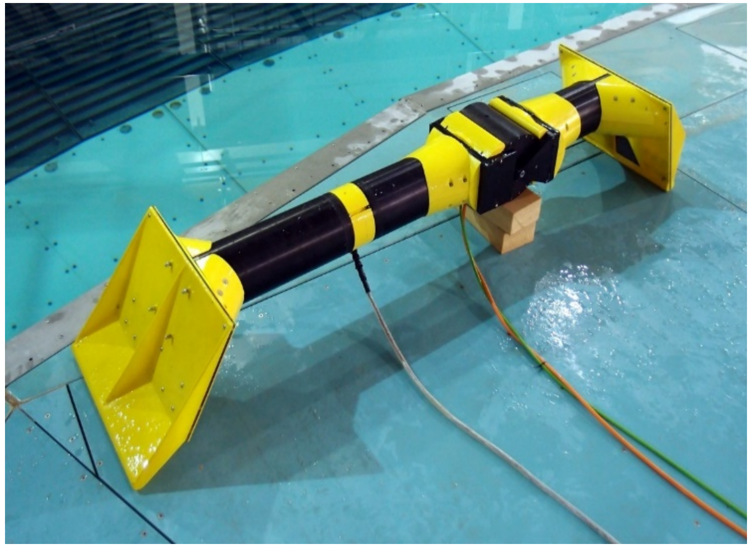


Fig. Blue Star 1/20th scale by Mocean

- Linear Noncausal Optimal Controller (LNOC) was implemented.
- Deterministic Sea Wave Prediction (DSWP) technique was used to predict incoming waves.

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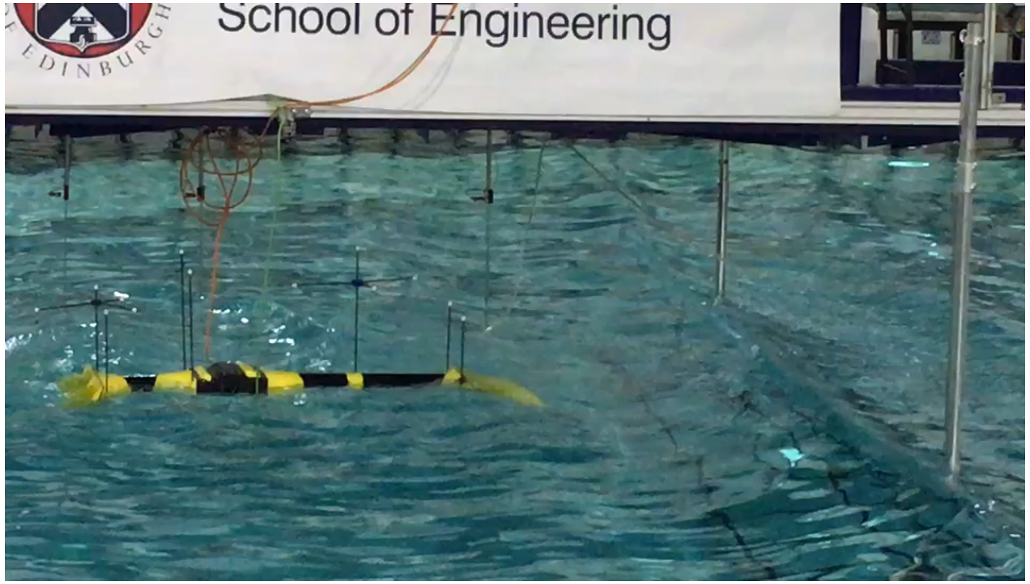

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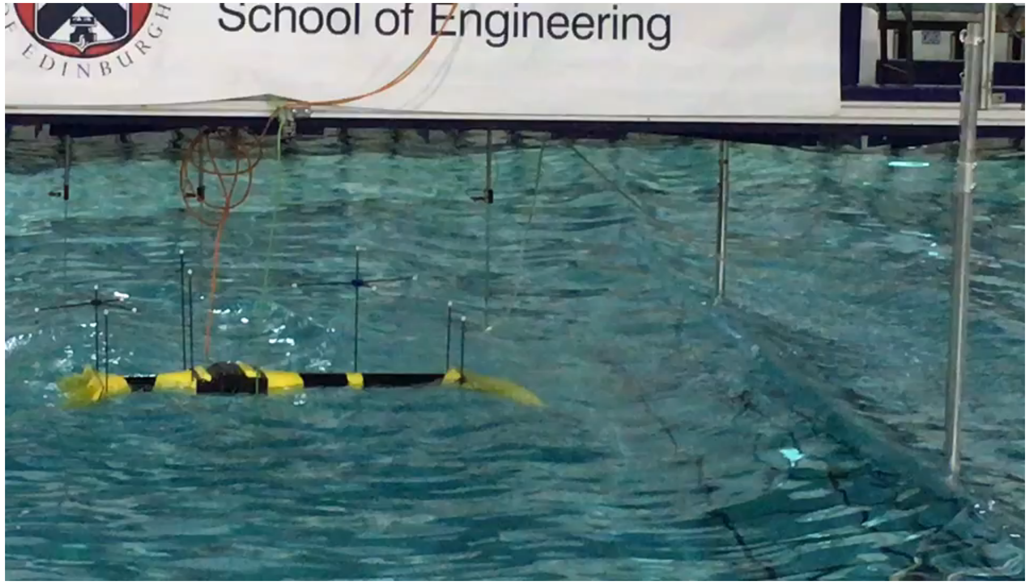

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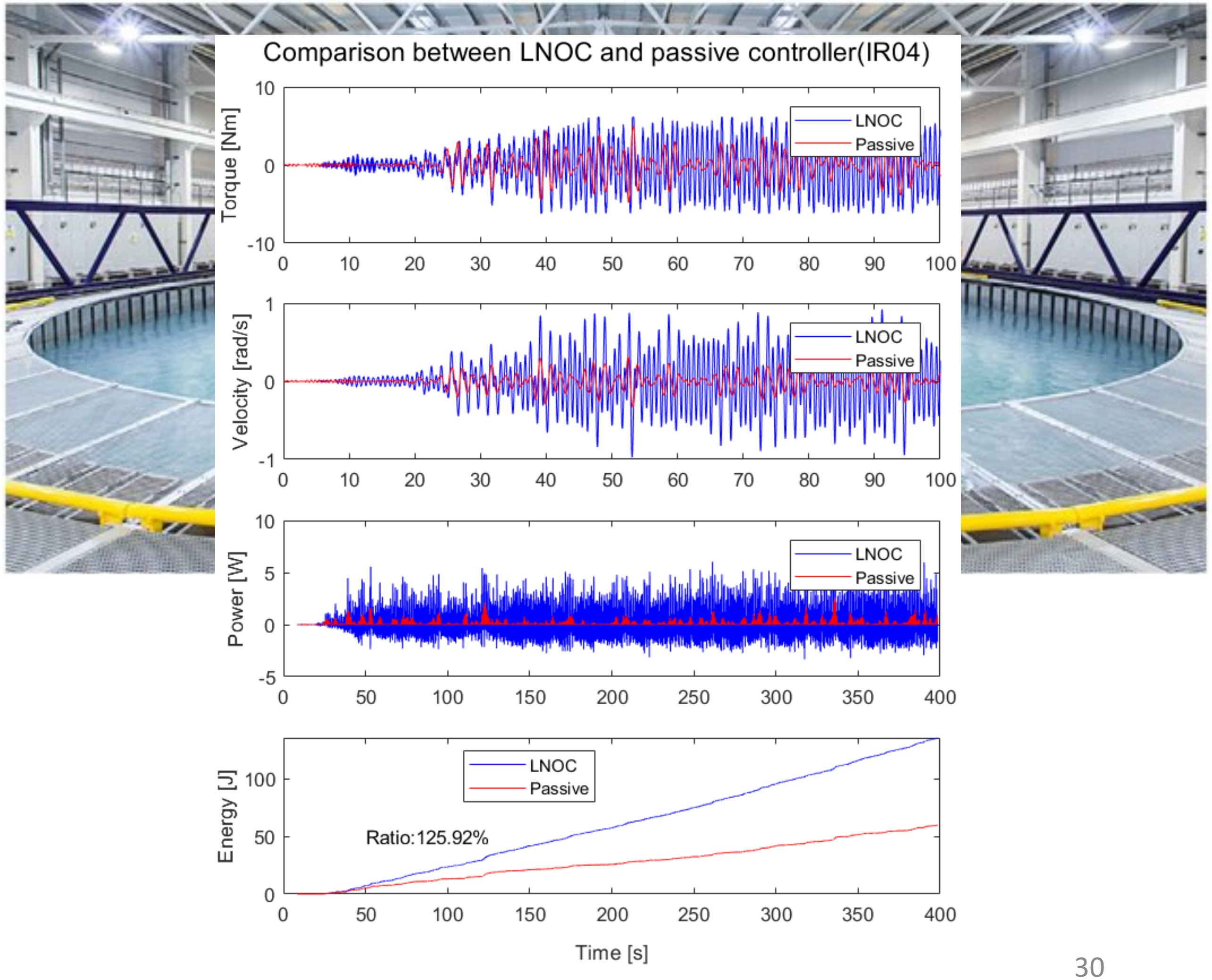

Mocean WEC at Sea Trial

Wave Energy Topic 4 – Onshore Wave Energy

- Collaboration with: Eco Wave Power Ltd.
- The first company built up onshore wave energy power station at Gibraltar and Israel.

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Innovate UK

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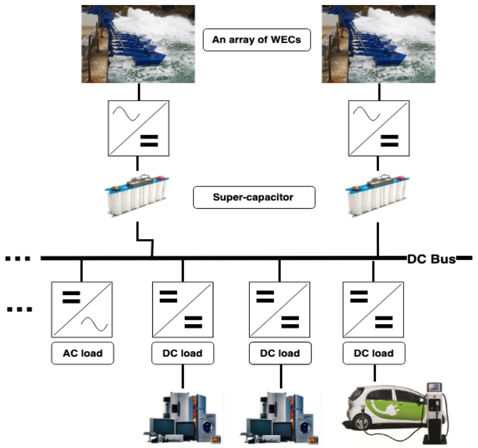
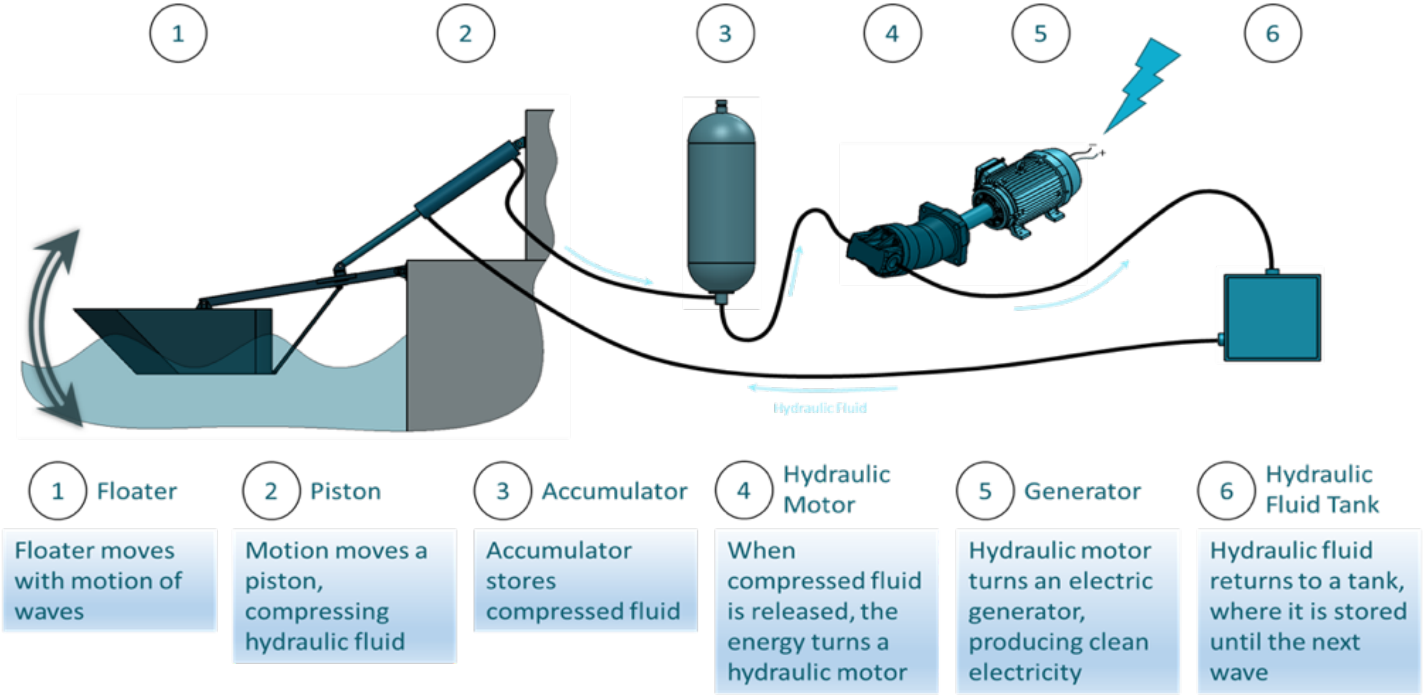
BRITISH COUNCIL

Microgrid Powered by Wave for Islands

- **Aim:** Develop a microgrid powered by waves to provide clean electricity for rural coastal areas or remote islands.





Wave Energy Topic 5 – Wave energy farm control

Wave Energy Farm Control

New control challenges in wave farm:

- Strong wave coupling between WECs.
- Farm modelling complexities.
- Large computational load for control.

The wave farm control objectives:

- Coordinate devices,
- Maximise farm energy output,
- Improve power quality.
- Maintain safe operations,
- Distribute computational load.

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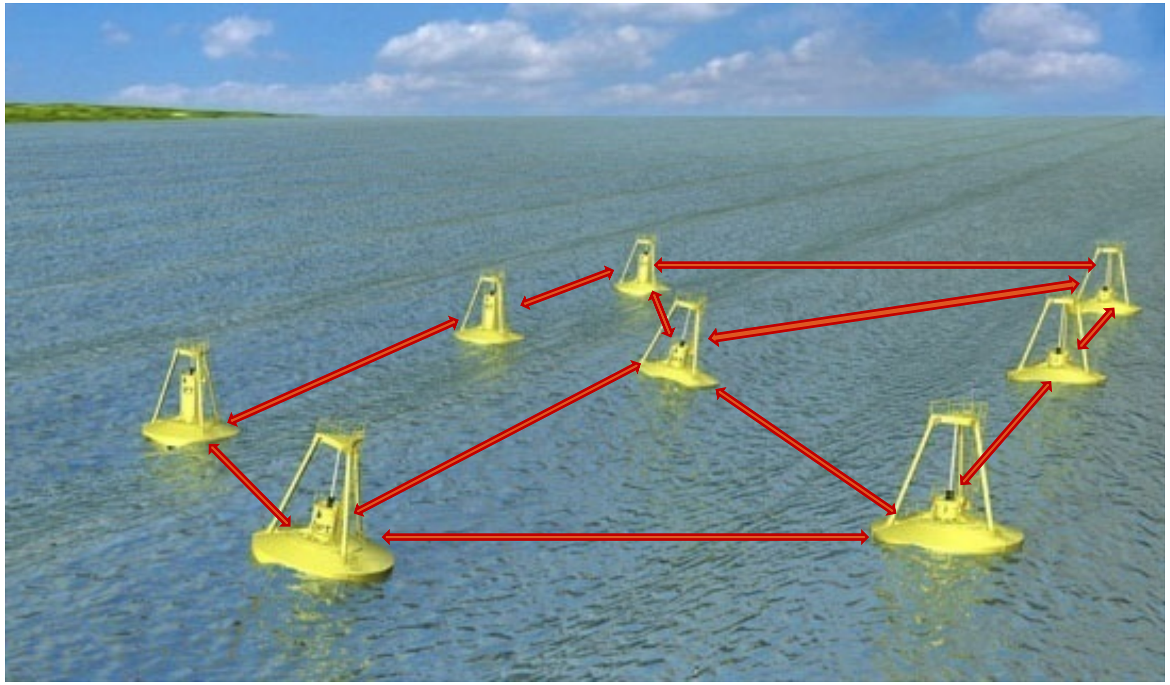
Wave Energy Farm Control



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Candidate MPC strategies

1. Decentralized MPC: Ignore the WEC interactions and control each WEC separately. But control performance degrades for strong coupling cases.

2. Centralized MPC: Apply MPC to the model for the whole array. But the computational burden can be too high to be tractable for large number of WECs.

3. **Distributed MPC:** WECs are controlled by local controllers, which coordinate with each other to approximate the optimal solution.

Benefits:

Distributed computational burden.

Approximated global optimal control.

Distributed MPC

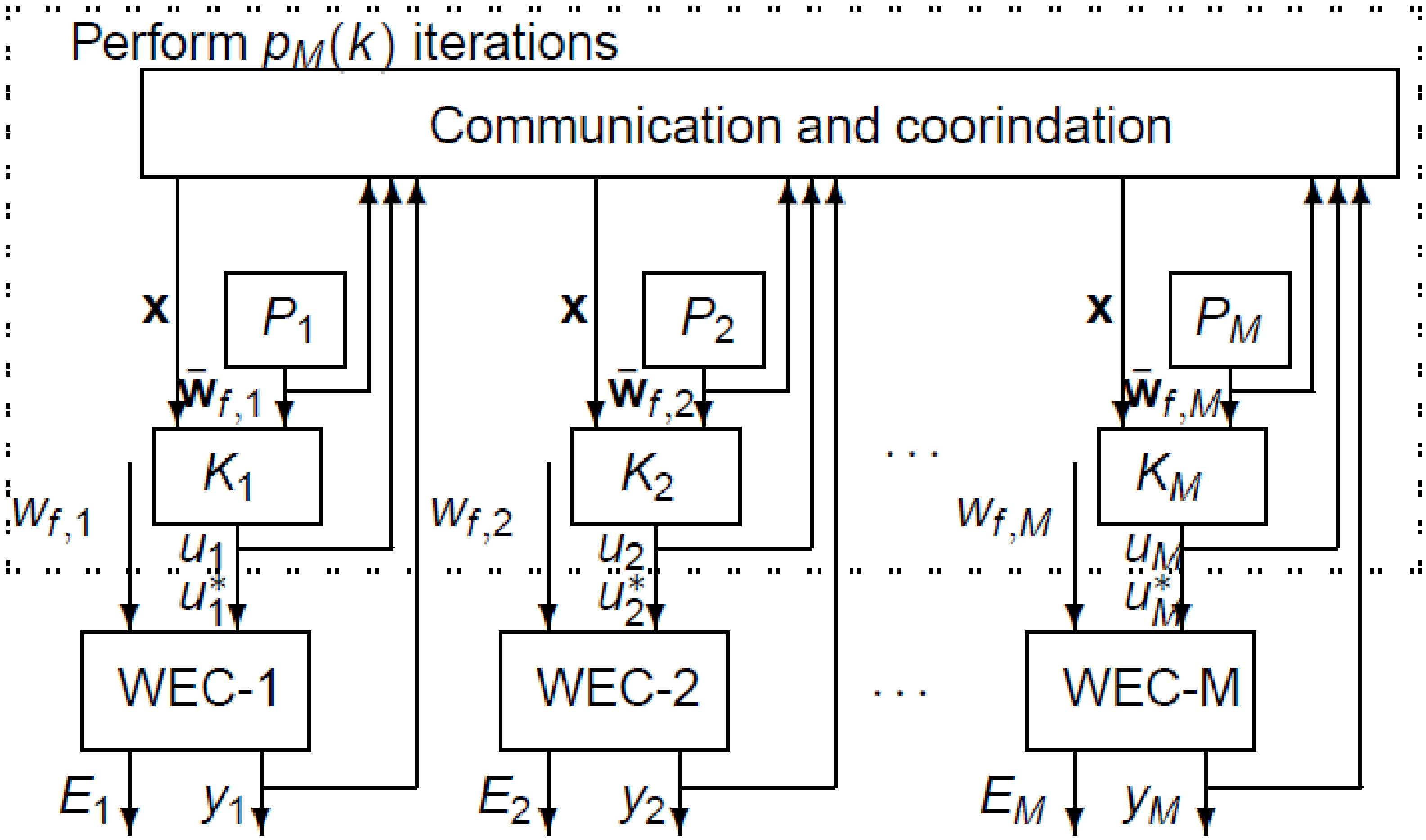
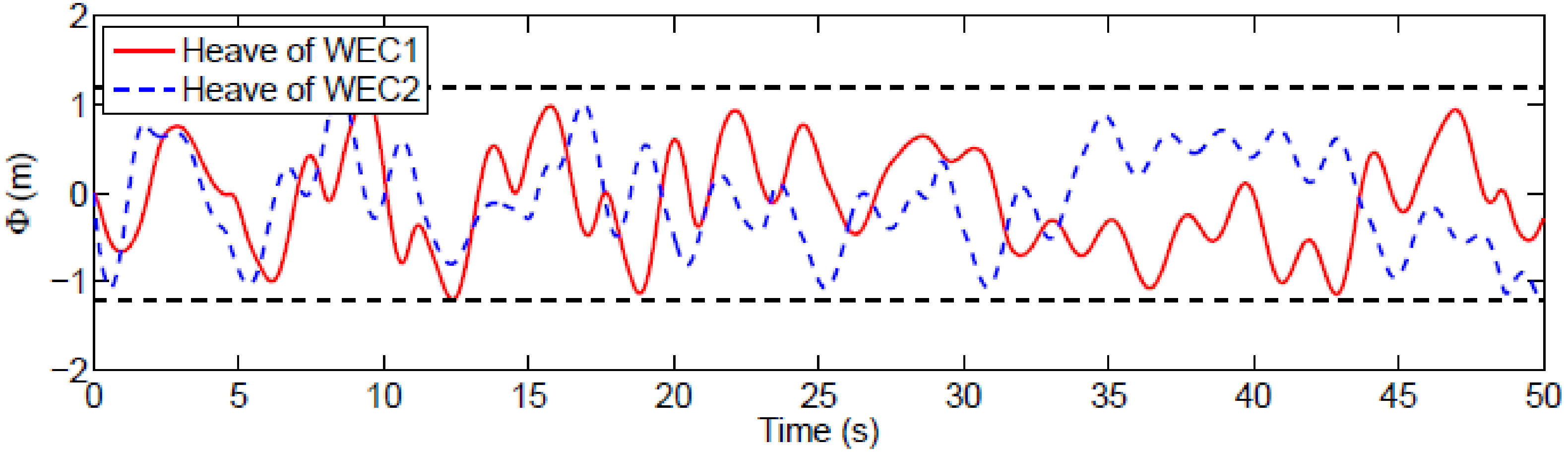
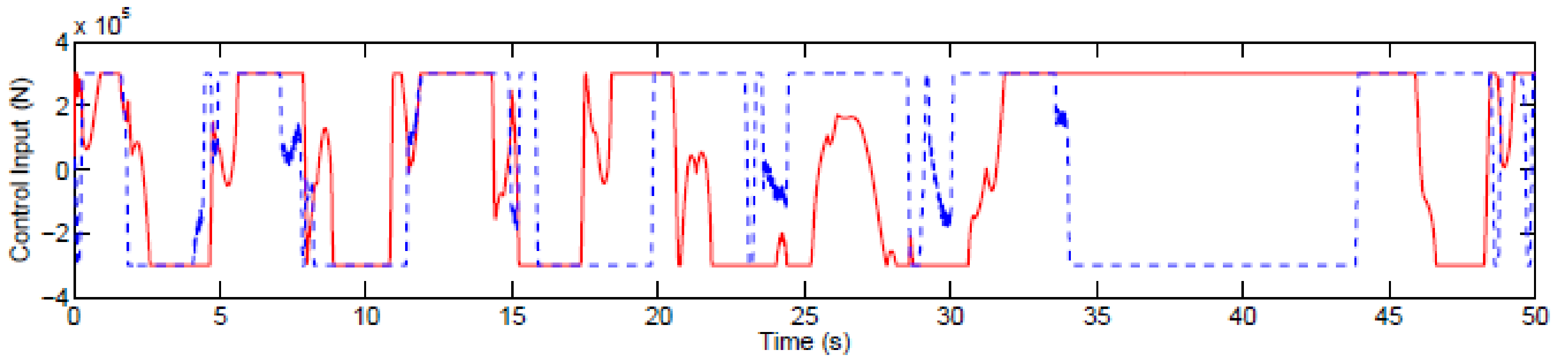


Figure : Distributed MPC of an array of WECs

Simulation – constraint satisfaction



(a) Constrained state



(b) Control input

Simulation – energy output