EMS717U/EMS717P

Renewable Energy Sources

Wave energy

Wave energy

Content

- **Origin**
- **Potential**
- **Characteristics of wave:** velocity, amplitude, wavelength, period
- **Principles of conversion**
- **Sample cases**

Wave: Physical principles

Ocean waves are generated by wind passing over stretches of water. The precise mechanisms involved in the interaction between the wind and the surface of the sea are complex and not yet completely understood. Three main processes appear to be involved.

1. Initially, air flowing over the sea exerts a tangential stream on the water surface, resulting in the formation and growth of waves.

2. Turbulent air flow close to the water surface creates rapidly varying shear stresses and pressure fluctuations. Where these oscillations are in phase with existing waves, further wave development occurs.

3. Finally, when waves have reached a certain size, the wind can exert a stronger force on the up-wind face of the wave, causing additional wave growth.

Wave

Physical characteristics: velocity, amplitude, wavelength, period

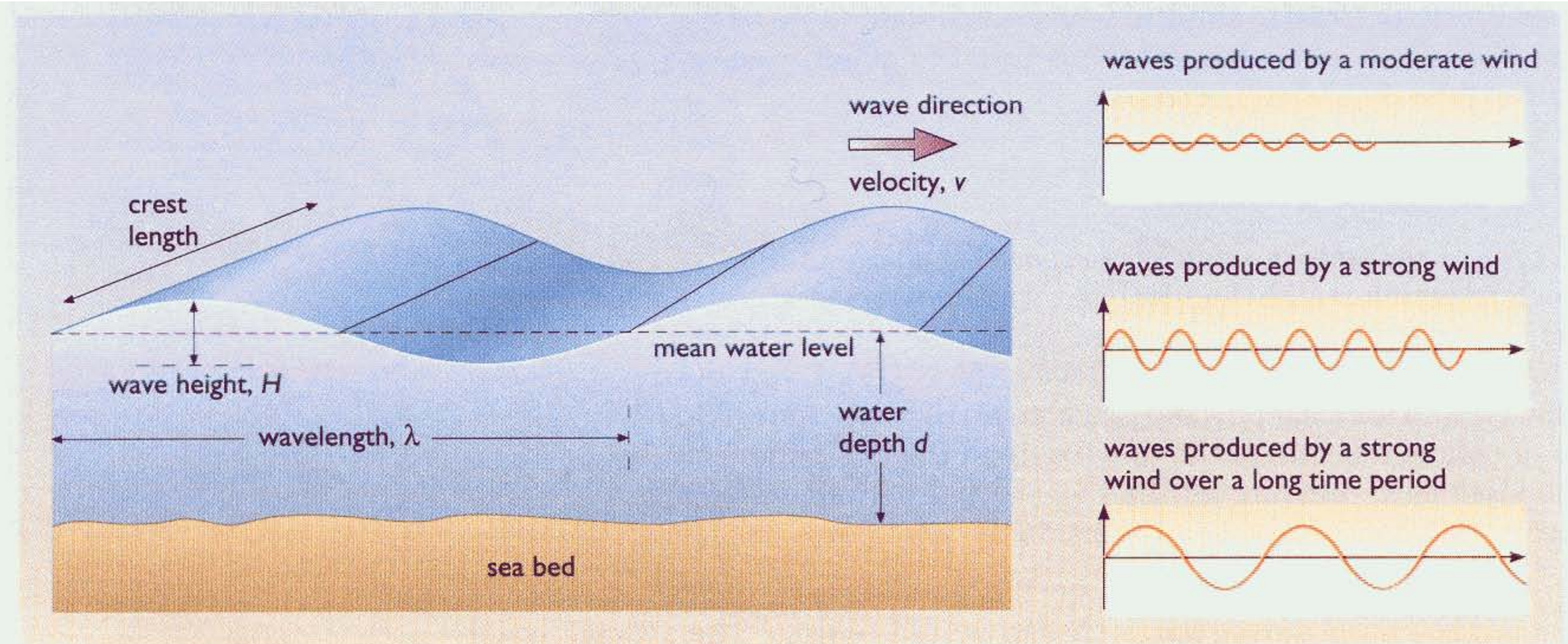


Figure 8.5 Characteristics of an idealized wave

Wave: calculation of power

Box 8.3 Wave characteristics and wave power

The shape of a typical wave is described as **sinusoidal** (that is, it has the form of a mathematical *sine* function). The difference in height between peaks and troughs is known as the **height**, H, and the distance between successive peaks (or troughs) of the wave is known as the **wavelength**, λ.

Suppose that the peaks and troughs of the wave move across the surface of the sea with a velocity, v. The time in seconds taken for successive peaks (or troughs) to pass a given fixed point is known as the **period**, T. The **frequency**, ν, of the wave describes the number of peak-to-peak (or trough-to-trough) oscillations of the wave surface per second, as seen by a fixed observer, and is the reciprocal of the period. That is, $\nu = 1 / T$.

If a wave is travelling at velocity v past a given fixed point, it will travel a distance equal to its wavelength λ in a time equal to the wave period T. So the velocity v is equal to the wavelength λ divided by the period T, i.e.:

$$v = \lambda / T$$

The power, P, (in kilowatts per metre) of an idealized ocean wave is approximately equal to the square of the height, H (metres), multiplied by the wave period, T (seconds). The exact expression is the following:

$$P = \frac{\rho g^2 H^2 T}{32\pi}$$

where P is in units of watts per metre, ρ is the density of water and g is the acceleration due to gravity (9.81 m s^{-2}).

Deep water waves

If the depth of water is greater than about half of the wavelength λ, the velocity of a long ocean wave can be shown to be proportional to the period as follows:

$$v = \frac{gT}{2\pi}$$

This leads to the useful approximation that the velocity in metres per second is about 1.5 times the wave period in seconds.

An interesting consequence of this result is that in the deep ocean the long waves travel faster than the shorter waves.

If both the above relationships hold, we can find the deep water wavelength, λ, for any given wave period:

$$\lambda = \frac{gT^2}{2\pi}$$

Intermediate depth waves

As the water becomes shallower, the properties of the waves become increasingly dominated by water depth. When waves reach shallow water, their properties are completely governed by the water depth, but in intermediate depths (i.e. between $d = \lambda / 2\pi$ and $d = \lambda / 4\pi$) the properties of the waves will be influenced by both water depth d and wave period T.

Shallow water waves

As waves approach the shore, the seabed starts to have an effect on their speed, and it can be shown that if the water depth d is less than a quarter of the wavelength, the velocity is given by:

$$v = \sqrt{g d}$$

In other words, the velocity under these conditions is equal to roughly three times the square root of the water depth d – it no longer depends on the wave period.

Wave: statistical distribution

Leads to wave rose similar to wind rose

Table 8.1 North Atlantic offshore wave conditions

	period / s	amplitude / m	power density / $kW\ m^{-1}$	velocity / $m\ s^{-1}$	wavelength / m
storm	14	14	1700	23	320
average	9	3.5	60	15	150
calm	5.5	0.5	1	9	50

Wave: statistical distribution

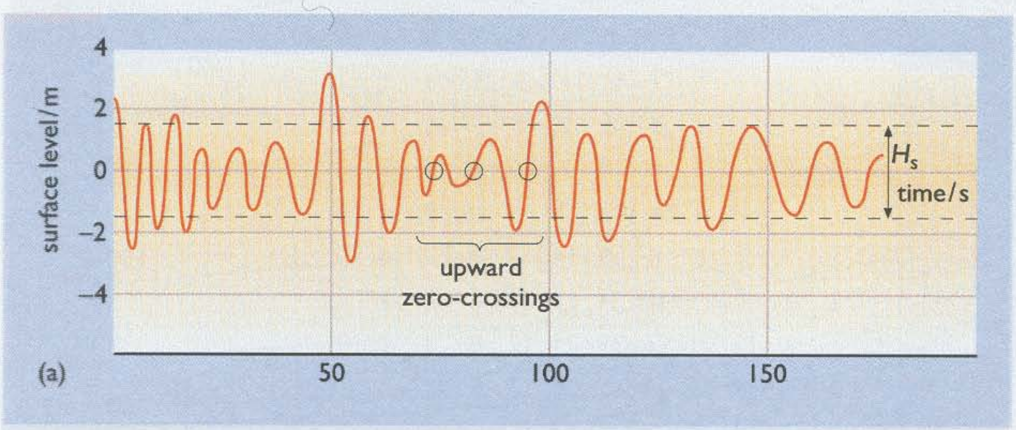
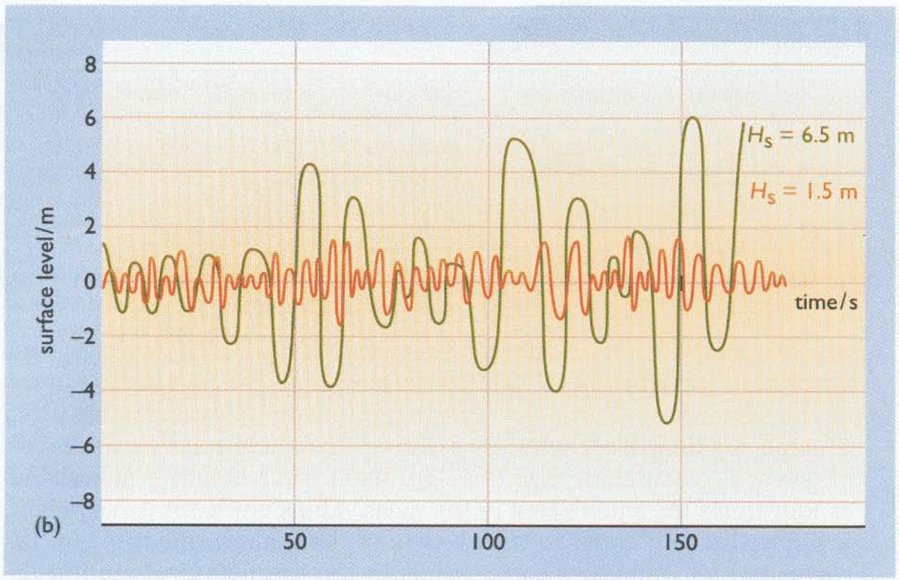


Figure 8.6 (a) A typical wave record. In this example the significant wave height H_s = 3 m. The successive upward movements of the surface are indicated with small circles. In this case there are 15 crossings in 150 seconds, so T_e = 10 seconds. From this, P (kW m^{-1}) = (3^2 × 10) / 2 (kW m^{-1}) = 45 kW m^{-1}

- **one day**

(b) Two wave records are shown here for the same location but represent recordings taken on different days



- **different days**

Wave: statistical distribution

Scatter diagram and wave rose

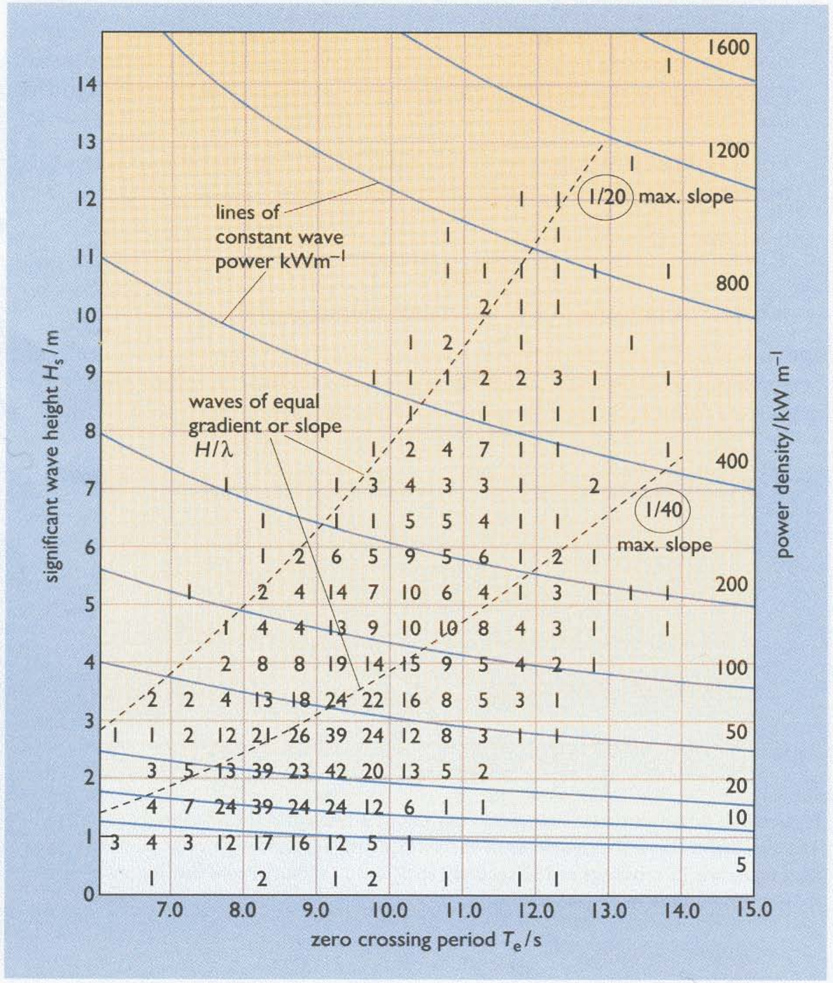


Figure 8.7 Scatter diagram of significant wave height (H_s) against zero crossing period (T_e) for 58°N 19°W in the north Atlantic. The numbers on the graph denote the average number of occurrences of each H_s and T_e in each 1000 measurements made over one year. The most frequent occurrences are at $H_s \sim 2$ m, $T_e \sim 9$ s

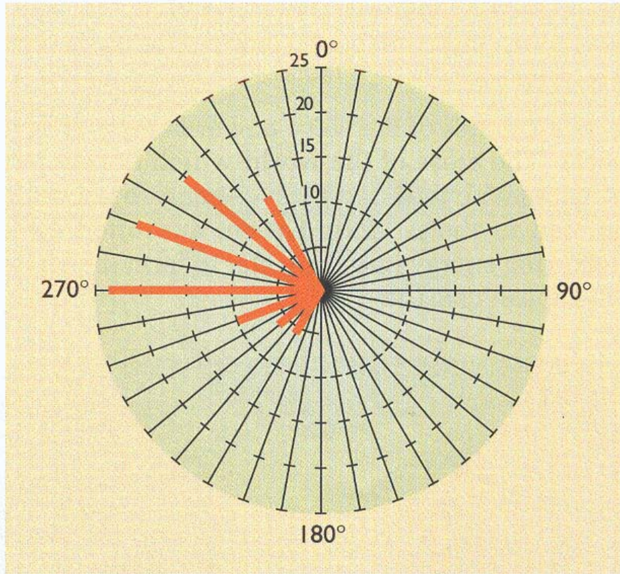


Figure 8.9 A directional rose for waves. The length of the line in each sector represents the average annual power in that sector. In this case most of the waves are coming from the west (Source: Thorpe, 1999)

Wave power: world capacity

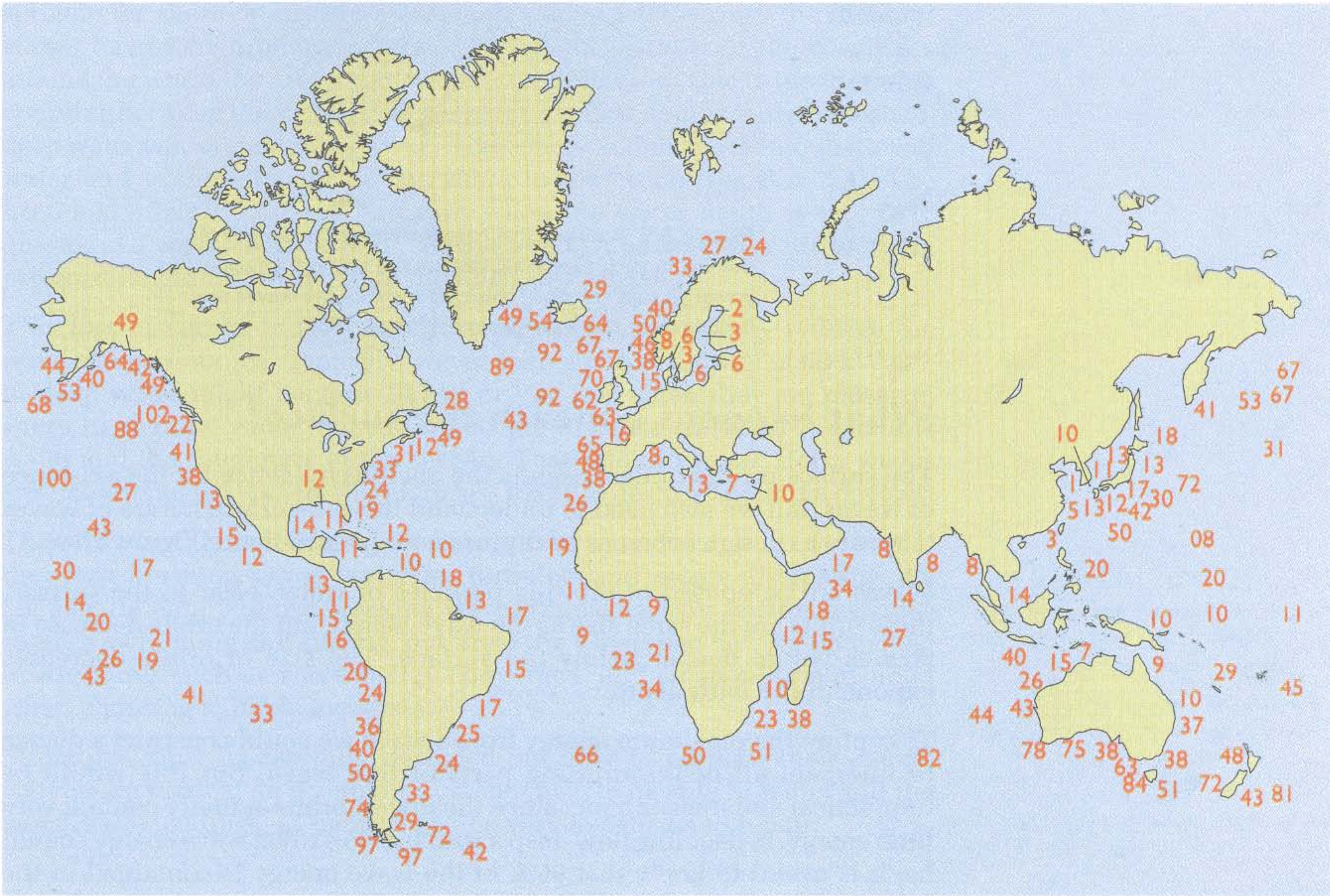


Figure 8.8 Annual average wave power in kilowatts per metre (kW m^{-1}) of crest length, for various locations around the world

Wave power

Effect of water depth

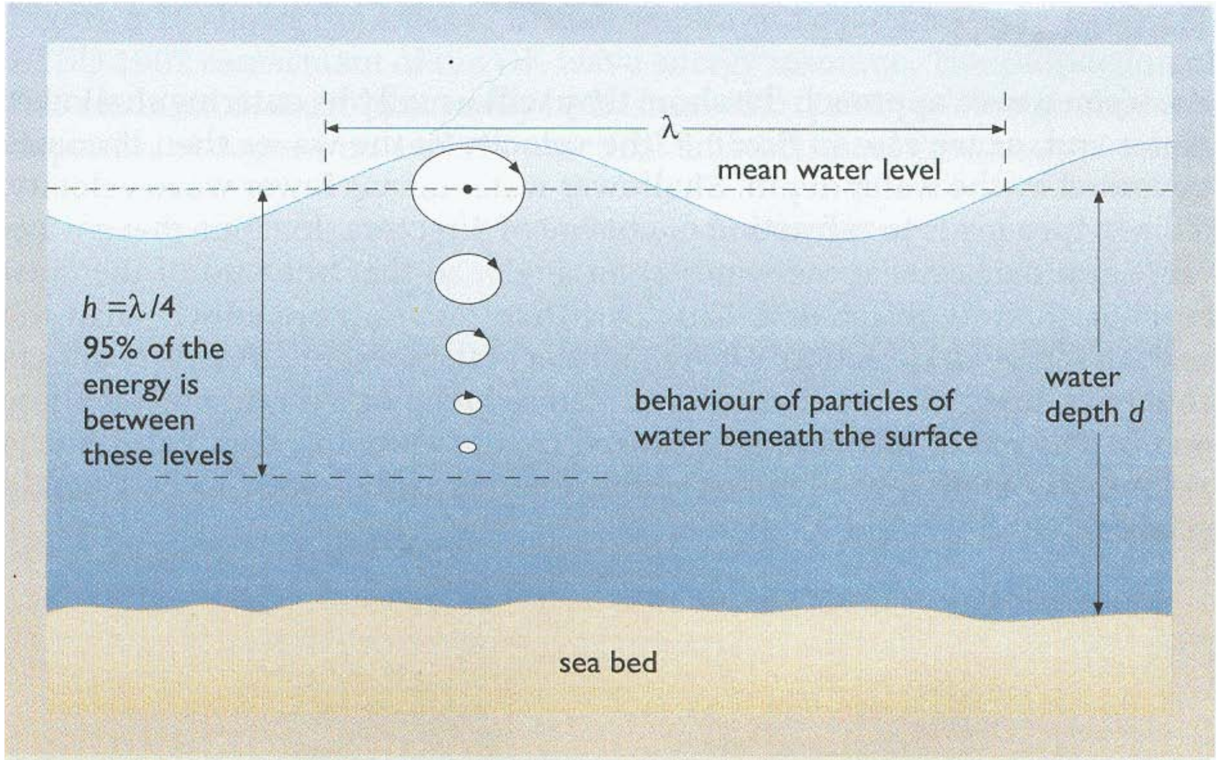


Figure 8.10 Behaviour of particles of water beneath the surface

Wave power: UK view

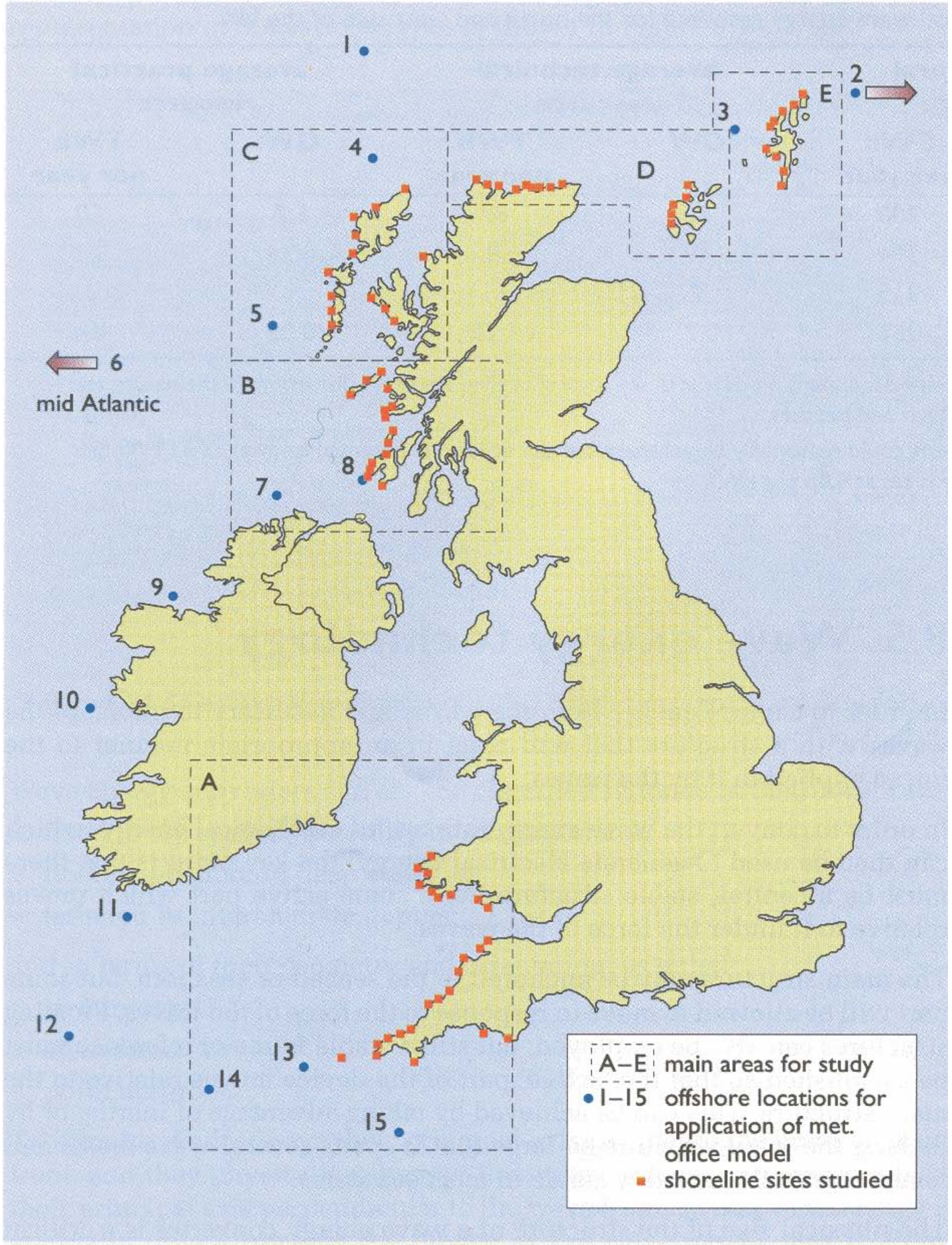


Figure 8.13 Wave power sites studied in the ETSU review of wave energy around the UK

Harnessing wave power

Case 1: TAPCHAN (TAPered CHANnel)

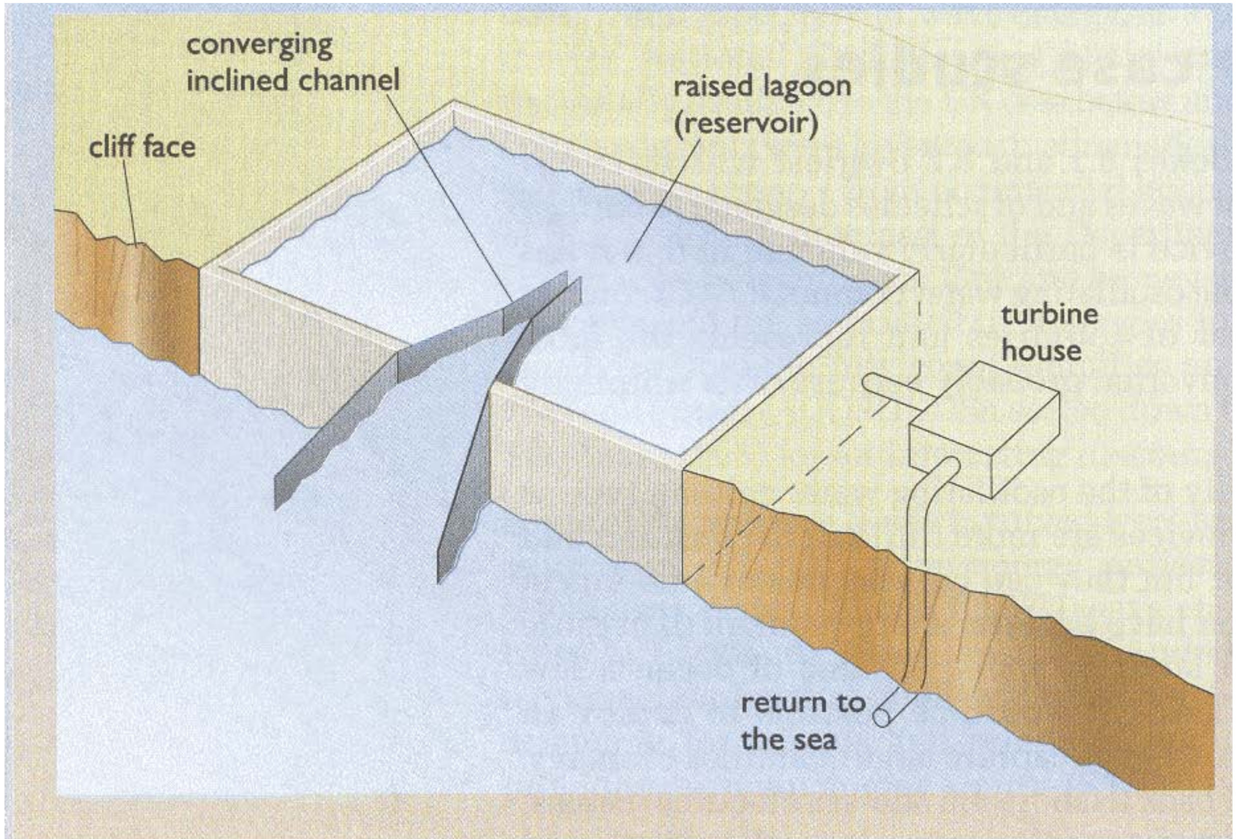


Figure 8.1 (a) The tapered channel wave energy conversion device: TAPCHAN

Harnessing wave power

Case 2: Oscillating water column (OWC)

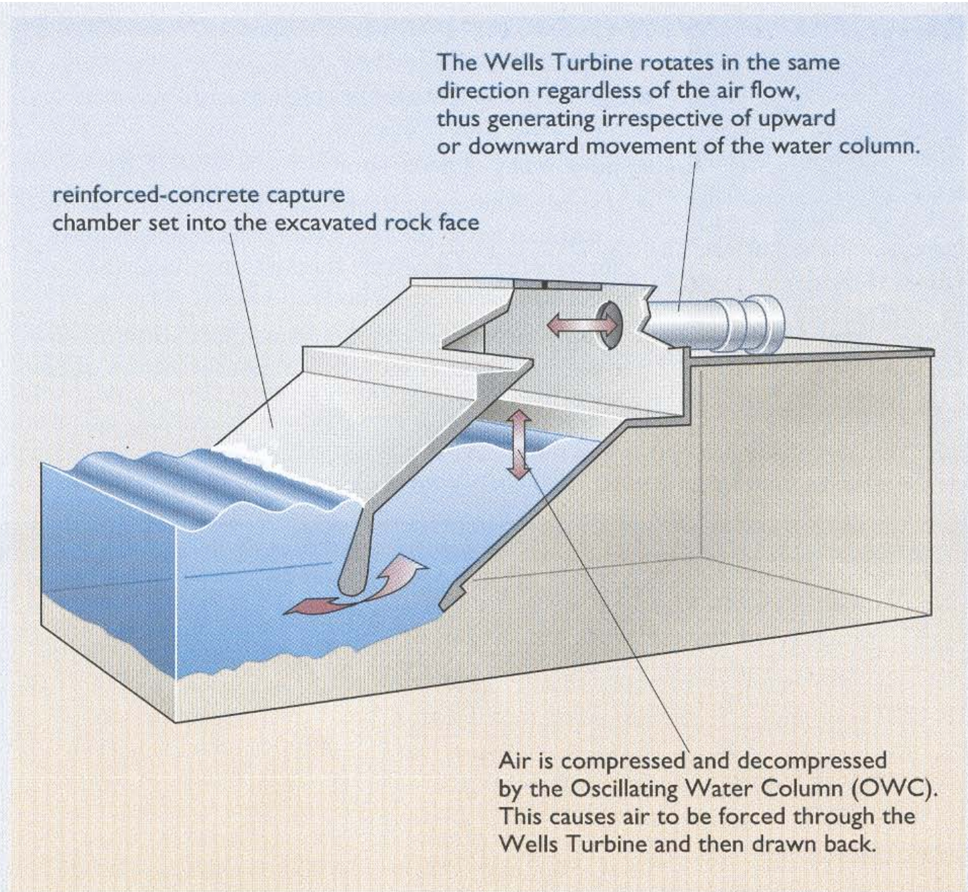
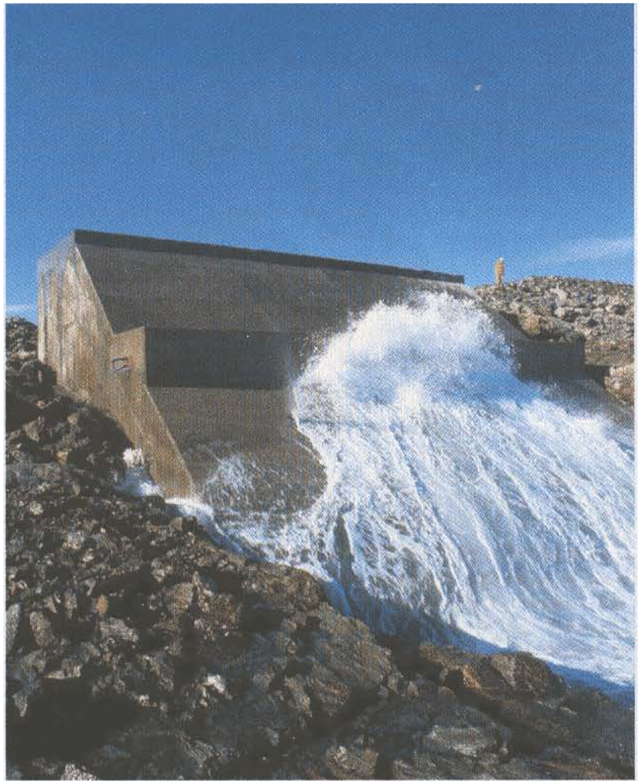


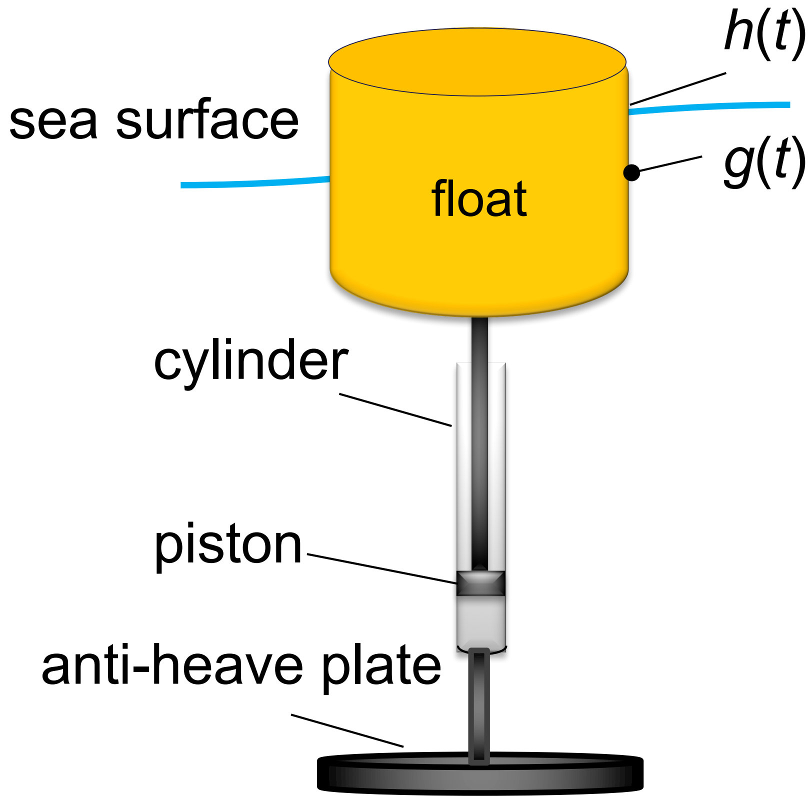
Figure 8.3 Outline of the LIMPET device on Islay



Harnessing wave power

Case 3: A point absorber

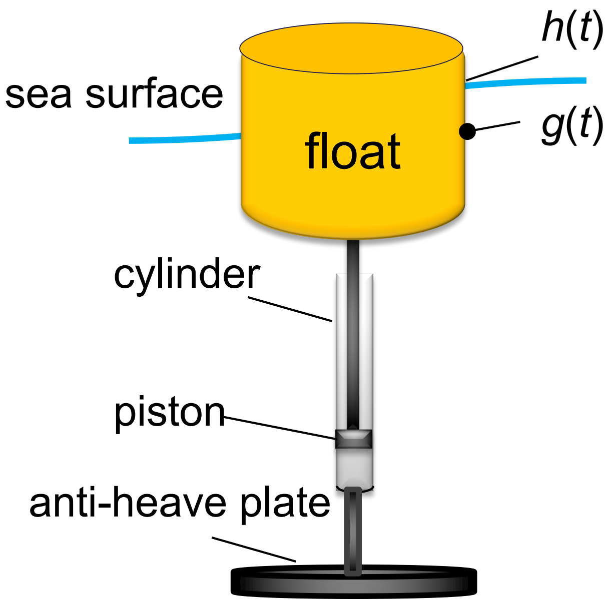
Power Buoy developed by
Ocean Power Technology (OPT)



Schematic Diagram

Working principle of point absorber control

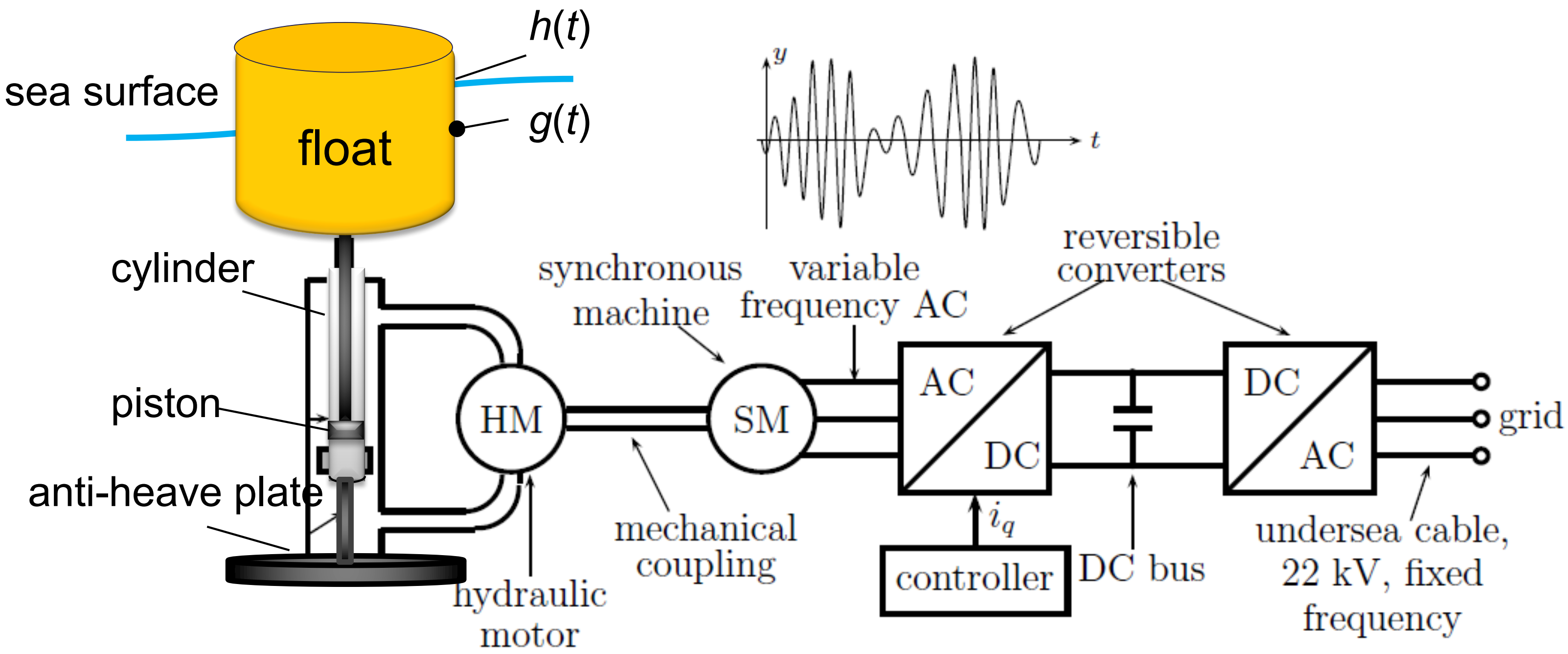
Case 3: A point absorber



Schematic Diagram

Working principle of point absorber control

Case 3: A point absorber



Schematic Diagram

The point absorber model

Case 3: A point absorber

The dynamic model can be represented as

$$m\ddot{y} = f_c + f_h + f_r + f_{ext} \qquad (1)$$

where m – float mass;

y – vertical displacement of the float.

f_c – control force;

f_{ext} – wave excitation force;

f_h – buoyancy force, $f_h = -k_h y$;

f_r – radiation force,

$f_r(t) = -\mu_\infty \ddot{y} - \int_{-\infty}^{t} k_r(t-\tau)\dot{y}(\tau)d\tau$ (Cummins equation)

here μ_∞ is the "added mass", representing the force from instantaneous motion, and the convolution represents the forces due to the transient motion. We can use a LTI model to represent the convolution as

$$\dot{x}_r(t) = A_r x_r(t) + B_r \dot{y}(t)$$
$$y_r(t) = C_r z_r(t) = \int_{-\infty}^{t} k_r(t-\tau)\dot{y}(\tau)d\tau \qquad (2)$$

Optimal Control Formulation

Case 3: A point absorber

- Control objective: maximize wave energy extraction while maintaining the safe operation.
- Constrained optimal control problem:

$$\max_{\mathrm{u(t)}} \int_0^T P(t)\,dt$$

subject to: $\dot{x} = f(x, u, w, t)$ – WEC dynamics

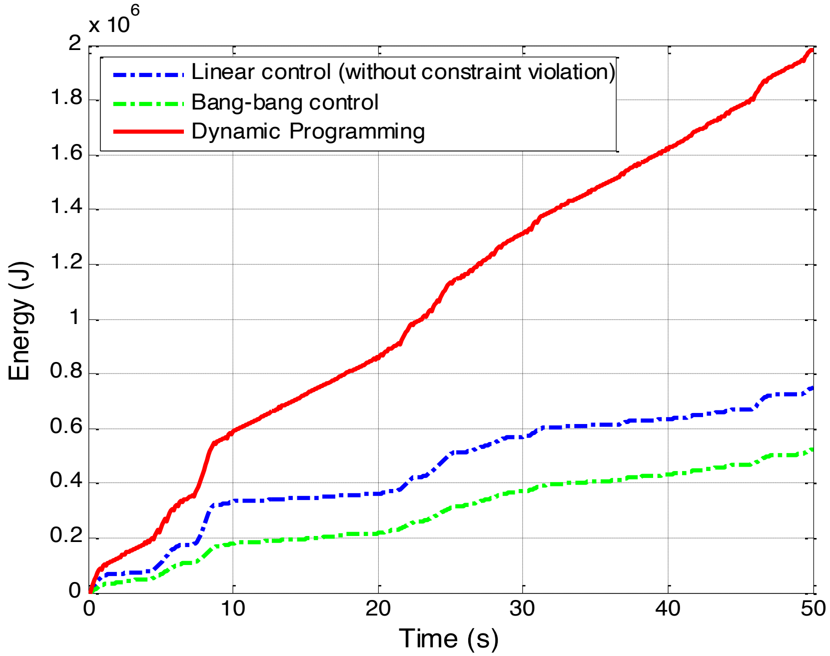
$u_{\min} \leq u(t) \leq u_{\max}$ – Actuator constraint

$z_{\min} \leq z(t) \leq z_{\max}$ – Safety constraint

Li G and Belmont MR (2014). **Model predictive control of sea wave energy converters–Part I: A convex approach for the case of a single device.** *Renewable Energy* vol. 69, 453-463.

Our control strategy can double the energy output!

Case 3: A point absorber



40 KW

- First realistically feasible WEC control strategy.
- Naturally integrate sea wave prediction.

15 KW

10 KW

Figure: Energy comparison when a scaled model is controlled by different methods.

Li G et al. (2012). **Wave energy converter control by wave prediction and dynamic programming.** *Renewable Energy,* vol. 48, 392-403.

Harnessing wave power

Case 3: A point absorber

Wave Energy Farm Control

Case 3: A point absorber

New control challenges in wave farm:

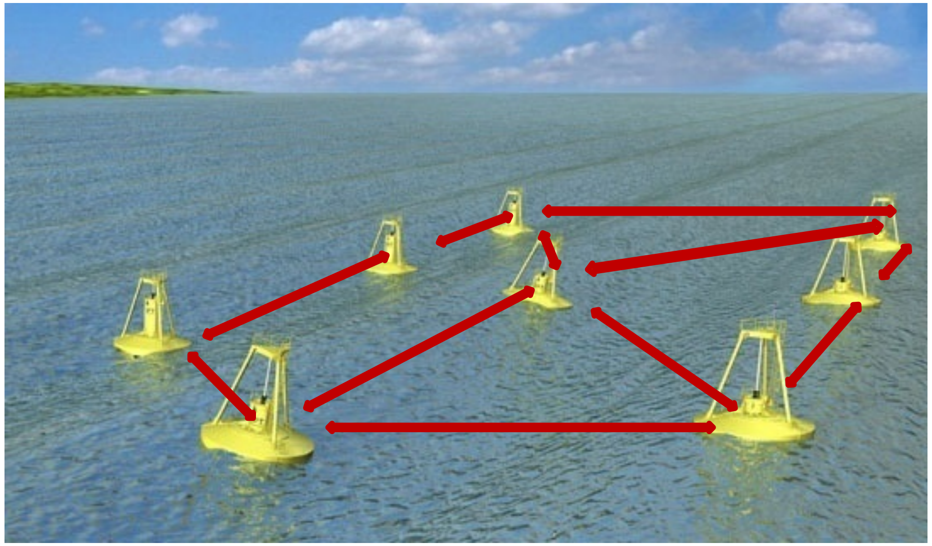
- Strong wave coupling between WECs.
- Farm modelling complexities.
- Large computational load for control.

The wave farm control objectives:

- Coordinate devices,
- Maximise farm energy output,
- Improve power quality.
- Maintain safe operations,
- Distribute computational load.

Wave Energy Farm Control

Case 3: A point absorber



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Harnessing wave power

Case 4: Pelamis wave energy converter

Pelamis Wave Energy Converter

Harnessing wave power

Case 5: Edinburgh Duck

Edinburgh Duck