Maths & Stats Pre-Sessional Tutorial

Solutions to Additional Exercises for topics 2 and 3

Exercise 1

a) What is the probability that the mutual fund outperforms the market, and its manager did not graduate from a top 20 MBA programme?

Solution: This is a joint probability. It is P(Y1 and X2) = 0.06

b) What is the probability that a mutual fund does not outperform the market and its manager graduated from a top 20 MBA programme?

Solution: This is a joint probability. It is P(Y2 and X1) = 0.29

c) Calculate the marginal probabilities.

Solution: Marginal probabilities are:

P(X1) = 0.11 + 0.29 = 0.4

P(X2) = 0.06 + 0.54 = 0.6

P(Y1) = 0.11 + 0.06 = 0.17

P(Y2) = 0.29 + 0.54 = 0.83

d) What is the probability that mutual fund managers did not graduate from a top 20 MBA programme?

Solution: This is a marginal probability. It is P(X2) = 0.6

e) Suppose now that we select one mutual fund at random and discover that it did not outperform the market. What is the probability that a graduate of a top 20 MB a programmer manages it?

Solution: This is a conditional probability. It is: P(X1|Y2) = $\frac{\mathrm{P(X1and\ Y2)}}{P(Y2)} = \frac{0.29}{0.83} = 0.349$

Exercise 2 - Solution:

We first write the null and alternative hypothesis.

$$H_0: \mu = 100$$

$$H_1: \mu \neq 100$$

We then calculate the z-statistic: $z - stat = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} = \frac{95.54-100}{15/\sqrt{36}} = -1.74$

This will be a two-tailed test. We reject the null hypothesis if the |test statistic| is greater than the critical value. Be careful the test statistic is expressed in absolute value.

a) Using a 1% significant level, our critical values of z are therefore the two values that span the middle 95% of the area under the standard normal distribution. This means that the areas in each of the two tails is 2.5%. The critical values ($-z\alpha/_2$and $z\alpha/_2$) are -2.58 and 2.58.

   Therefore, we can conclude that the null hypothesis is failed to be rejected. In other words, we can say that our drug did not have a significant effect on IQ.

b) Using a 10% significant level, the critical values ($-z\alpha/_2$and $z\alpha/_2$) are -1.645 and 1.645.

   Therefore, we can conclude that the null hypothesis is rejected.

Exercise 3 - Solution:

We first write the null and alternative hypothesis.

$$H_0: \mu = 210$$

$$H_1: \mu > 210$$

We then calculate the z-statistic: $z - stat = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} = \frac{212.79-210}{8.5/\sqrt{42}} = 2.13$

This will be a one-tailed test. We reject the null hypothesis if the test statistic is greater than the critical value.

a) Using a 5% significant level, the critical value z_{α}) is 1.645.

We can conclude that the null hypothesis is rejected. Therefore, new mean significantly greater than the desired mean of 210.

b) We want to calculate P(Z > 2.13)

P(Z > 2.13) = 1 – P(Z < 2.13) = 1 – 0.98341 = 0.01659

This probability is smaller than 0.05

Exercise 4

Consider a random variable X. X is normally distributed, X ~ N(μ, σ^2) = N(40, 36)

a) Find P(X > 50)
b) Find P(X < 45)
c) Find P(31 < X < 45)

Solution:

a) The Z-score is $\frac{X-\mu}{\sigma} = \frac{50-40}{\sqrt{36}} = 1.67.$

Therefore we want to find P(Z > 1.67) = 1 - P(Z < 1.67) = 1 – 0.95254 = 0.04746

b) The Z-score is $\frac{X-\mu}{\sigma} = \frac{45-40}{\sqrt{36}} = 0.83.$

Therefore we want to find P(Z < 0.83) = 0.79673

c) The Z-scores are $\frac{X-\mu}{\sigma} = \frac{45-40}{\sqrt{36}} = 0.83$ and $\frac{X-\mu}{\sigma} = \frac{31-40}{\sqrt{36}} = -1.50$

Therefore we want to find P(-1.5 < Z < 0.83) = P(Z < 0.83) - P(Z < -1.5) = 0.79673 – 0.06681 = 0.72992