Maths & Stats Pre-Sessional Tutorial

Topic 3: Estimation and Hypothesis Testing

Exercise 1

You invest £3000 in one stock and your partner invests £2000 in another. Over the next year, each pound invested in your stock will increase by X pounds and each pound invested in your partner's will increase by Y pounds. X and Y are random variables with the following properties:

- X has a mean of 0.09 and a standard deviation of 0.20
- Y has a mean of 0.12 and a standard deviation of 0.27
- The correlation between X and Y is 0.06

Your individual earnings are 3X thousand, your partner's individual earnings are 2Y thousand and your family earnings are the sum of the two.

a) What is the expected value of your family earnings?

b) What is the standard deviation of your family portfolio earnings?

Exercise 2

Given a population with a mean of μ = 100 and a variance of σ^2 = 900, the central limit theorem applies when the sample size is n ≥ 25. A random sample of size n = 30 is obtained.

a) What are the mean and variance of the sampling distribution for the sample means?

b) What is the probability that $\bar{x}$ > 109?

c) What is the probability that 96 ≤ $\bar{x}$ ≤ 110?

d) What is the probability that $\bar{x}$ ≤ 107?

You can find the Standard Normal Distribution Tables at the end of this document.

Exercise 3

The mean selling price of flats in historic buildings in Manchester over a year was \$215,000. The population standard deviation was \$25,000. A random sample of 100 new unit sales was taken. The central limit theorem applies for $n \geq 30$.

a) What is the probability that the sample mean selling price was more than \$210,000?
b) What is the probability that the sample mean selling price was between \$213,000 and \$217,000?
c) What is the probability that the sample mean selling price was between \$214,000 and \$216,000?
d) Without doing the calculations, state in which of the following ranges the sample mean selling price is most likely to lie:
[213,000;215,000] [214,000;216,000] [215,000;217,000] [216,000;218,000]
e) Suppose that, after you had done these calculations, a friend asserted that the population distribution of selling prices of flats in historic buildings in Manchester was almost certainly not normal. How would you respond?

Exercise 4

In a random sample of 361 owners of small businesses that had gone into bankruptcy, 105 reported conducting no marketing studies prior to opening the business. Test the hypothesis that at most 25% of all members of this population conducted no marketing studies before opening their businesses. Assume that the population variance is 0.1875, and use $\alpha = 0.05$.

You can find the Standard Normal Distribution Tables at the end of this document.

Check your knowledge:

Test your knowledge with the following multiple-choice questions.

For each question, select the correct answer. Explain your decision.

Question 1

Which of the following is the branch of statistical inference?

a) Estimation
b) Hypothesis Testing
c) Both a) and b)
d) Neither a) nor b)

Question 2

The value of an estimator is called:

a) Expectation
b) Estimate
c) Variance
d) None of them

Question 3

Which of the following is not an assumption of parametric inference methods?

(a) Data is numeric.
(b) Population has a known distribution.
(c) Sample is sufficiently large.
(d) Data is categorical.

Question 4

A point estimator is defined as:

a) a range of values that estimates an unknown population parameter.
b) a range of values that estimates an unknown sample statistic.
c) a single value that estimates an unknown population parameter.
d) a single value that estimates an unknown sample statistic.

Question 5

Which of the following is not a characteristic for a good estimator?

a) Being unbiased.
b) Being consistent.
c) Being efficient.
d) All of these choices are true.

Question 6

An unbiased estimator of a population parameter is defined as:

a) an estimator whose expected value is equal to the parameter.
b) an estimator whose variance is equal to one.
c) an estimator whose expected value is equal to zero.
d) an estimator whose variance goes to zero as the sample size goes to infinity.

Question 7

An estimator is said to be consistent if:

a) it is an unbiased estimator.
b) the variance of the estimator is zero.
c) the difference between the estimator and the population parameter stays the same as the sample size grows larger.
d) the difference between the estimator and the population parameter grows smaller as the sample size grows larger.

Question 8

If there are two unbiased estimators of a population parameter available, the one that has the smallest variance is said to be:

a) a biased estimator.
b) efficient.
c) consistent.
d) unbiased.

Question 9

Let T_n be an estimator of θ. If E(T_n) = θ, then

(a) T_n is a sufficient estimator of θ
(b) T_n is an unbiased estimator of θ
(c) T_n is a consistent estimator of θ
(d) T_n is an efficient estimator of θ

Question 10

The bias of an estimator can be:

(a) Positive
(b) Negative
(c) Zero
(d) Any value

Question 11

Let $X_1, X_2, \dots X_n$ be a random sample of size *n* from a population. Then for the population variance σ^2

(a) $\frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2$ is an unbiased estimator.
(b) $\frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2$ is a biased estimator.
(c) $\sum_{i=1}^{n}(X_i - \bar{X})^2$ is an unbiased estimator.
(d) None of the above.

Question 12

The Central Limit Theorem states that the sampling distribution of the sample mean is approximately normal under certain conditions. Which of the following is a necessary condition for the Central Limit Theorem to be used?

a) The sample size must be large.
b) The population size must be large.
c) The population from which we are sampling must be normally distributed.
d) The population from which we are sampling must not be normally distributed.

Question 13

One year, the distribution of salaries for professional sports players had mean $1.6 million and standard deviation $0.7 million. Suppose a sample of 100 major league players was taken. Find the approximate probability that the average salary of the 100 players that year exceeded $1.1 million.

a) 0.7357
b) approximately 1
c) 0.2357
d) approximately 0

Question 14

Which statement best describes a parameter?

a) A parameter is a level of confidence associated with an interval about a sample mean or proportion.
b) A parameter is a numerical measure of a population that is almost always unknown and must be estimated.
c) A parameter is a sample size that guarantees the error in estimation is within acceptable limits.
d) A parameter is an unbiased estimate of a statistic found by experimentation or polling.

Question 15

You are considering moving to St, Albans, and you are concerned about the average one-way commute time. Does the average one-way commute time exceed 25 minutes? You take a random sample of 50 St. Albans residents and find an average commute time of 29 minutes with a standard deviation of 7 minutes.

a) $H_{0:}\ \mu = 25\ \ vs\ H_{1:}\ \mu > 25$
b) $H_{0:}\ \mu = 29\ \ vs\ H_{1:}\ \mu < 29$
c) $H_{0:}\ \mu = 25\ \ vs\ H_{1:}\ \mu \neq 25$
d) $H_{0:}\ \mu = 29\ \ vs\ H_{1:}\ \mu > 29$
e) $H_{0:}\ \mu = 25\ \ vs\ H_{1:}\ \mu < 25$

Question 16

Suzie has installed a new spam blocker program on her email. She used to receive an average of 20 spam emails a day. Is the new program working?

a) $H_{0:}\ \mu = 20\ \ vs\ H_{1:}\ \mu < 20$
b) $H_{0:}\ \mu > 20\ \ vs\ H_{1:}\ \mu = 20$
c) $H_{0:}\ \mu = 20\ \ vs\ H_{1:}\ \mu > 20$
d) $H_{0:}\ \mu = 20\ \ vs\ H_{1:}\ \mu \neq 20$
e) Not enough information is given.

Question 17

We reject the null hypothesis if the sample data falls in the

a) critical region
b) rejection region
c) acceptance region
d) None of these

Question 18

The statistic based on whose value the null hypothesis is rejected or not is called:

a) the test statistic
b) the critical value
c) both (a) and (b)
d) Neither (a) nor (b).

Question 19

The value of the test statistic which separates the rejection region and acceptance region is called:

a) test statistic value
b) level of significance
c) critical value
d) None of the above

Explanation: the critical values of a statistical test are the boundaries of the acceptance region of the test. The acceptance region is the set of values of the test statistic for which the null hypothesis is not rejected.

Question 20

In the test for proportion $H_{0:}\, p_1 = p_2 \;\; vs\; H_{1:}\, p_1 \neq p_2,$ the best critical region is given by

a) $Z > z_\alpha$
b) $Z > -z_\alpha$
c) $Z < z_\alpha$
d) None of the above

Question 21

A large university is interested in learning about the average time it takes students to travel to campus. The university sampled 238 students and asked each to provide the amount of time they spent traveling to campus. This variable, travel time, was then used conduct a test of hypothesis. The goal was to determine if the average travel time of all the university's students differed from 20 minutes. You know that the average travel time in your sample is 23.243 and the population standard deviation is 1.3133. Conducting a test at 1% significant level.

What conclusion can be made? When testing at α = 0.01…

a) …there is sufficient evidence to indicate that the average travel time of all students exceeds 20 minutes.
b) …there is sufficient evidence to indicate that the average travel time of all students is equal to 20 minutes.
c) …there is insufficient evidence to indicate that the average travel time of all students is equal to 20 minutes.
d) …there is insufficient evidence to indicate that the average travel time of all students exceeds 20 minutes.

Question 22

Repeat the exercise in question 17. Now conduct the test at 5% significant level.

What conclusion can be made? When testing at α = 0.05…

a) …there is sufficient evidence to indicate that the average travel time of all students exceeds 20 minutes.
b) …there is sufficient evidence to indicate that the average travel time of all students is equal to 20 minutes.
c) …there is insufficient evidence to indicate that the average travel time of all students is equal to 20 minutes.
d) …there is insufficient evidence to indicate that the average travel time of all students exceeds 20 minutes.

Question 23

A national organization has been working with utilities throughout the nation to find sites for large wind machines that generate electricity. Wind speeds must average more than 19 miles per hour (mph) for a site to be acceptable. Recently, the organization conducted wind speed tests at a particular site. Based on a sample of n = 45 wind speed recordings (taken at random intervals), the wind speed at the site averaged $\bar{x}$ = 19.9 mph, with a standard deviation of 4.5 mph. To determine whether the site meets the organization's requirements, consider the test, $H_{0:}\ \mu = 19\ \ vs\ H_{1:}\ \mu > 19$, where μ is the true mean wind speed at the site and α = 0.10. Suppose the value of the test statistic were computed to be 1.34. State the conclusion.

a) At α = 0.10, there is evidence to conclude the true mean wind speed at the site does not exceeds 19 mph.
b) At α = 0.10, there is evidence to conclude the true mean wind speed at the site exceeds 19 mph.
c) We don't have enough information to reach a conclusion.

Question 24

Statistical quality control is based on the theory of

a) probability
b) sampling
c) both (a) and (b)
d) neither (a) nor (b)

Question 25

The manager of a grocery store has taken a random sample of 100 customers. The average length of time it took the customers in the sample to check out was 2.9 minutes with a population standard deviation of 0.07 minutes. Do you reject the claim that mean waiting time of all customers is significantly less than 3 minutes. Use α = 0.05.

a) Yes. The mean waiting time of all customers is more than 3 minutes.
b) No. The mean waiting time of all customers is less than 3 minutes.

Appendix: tables

Table 1. Standard normal tables - 1

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08691	.08534	.08379	.08226
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.0	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.1	.46017	.45620	.45224	.44828	.44433	.44038	.43644	.43251	.42858	.42465
-0.0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414

Table 1. Standard normal tables - 2

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997