Queen Mary
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Examiner: Prof. O. Jenkinson

MTH6107 Chaos & Fractals
MID-TERM TEST
Date: 8th November 2023 *Time:* 10am

Complete the following information:

Name	
Student Number (9 digit code)	

The test has SIX questions. You should attempt ALL questions. Write your calculations and answers in the space provided. Cross out any work you do not wish to be marked.

Question	Marks
1	
2	
3	
4	
5	
6	
Total Marks	

Nothing on this page will be marked!

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Question 1.

Suppose $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = x^2 - 6$.

(a) Find all fixed points of f, and determine whether they are attracting or repelling.

(b) Determine the points of least period 2 for f.

[*25 marks*]

Answer 1.

Answer 1. (*Continue*)

Question 2.

Determine the basin of attraction for the fixed point 1 of the map $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2 - 2x + 2$.

[*15 marks*]

Answer 2.

Answer 2. (*Continue*)

Question 3.

Suppose $f : \mathbb{R} \to \mathbb{R}$ is C^1, and that $\{x_0, x_1, x_2, x_3, x_4, x_5\}$ is a 6-cycle with $f'(x_i) = (-2)^{i-3}$ for $0 \leq i \leq 5$. Determine, with justification, whether this cycle is attracting or repelling.

[*15 marks*]

Answer 3.

Answer 3. (*Continue*)

Question 4.

Suppose the diffeomorphism $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = 1 - x + \frac{1}{2}\cos x$. How many fixed points does f have? Justify your answer.

[*15 marks*]

Answer 4.

Answer 4. (*Continue*)

Question 5.

Determine those natural numbers n such that every continuous function $f : \mathbb{R} \to \mathbb{R}$ with an n-cycle also has an n^2-cycle.

[*15 marks*]

Answer 5.

Answer 5. (*Continue*)

Question 6.

Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^3$ and $g(x) = -x^3$. Are f and g topologically conjugate to each other? Justify your answer.

[*15 marks*]

Answer 6.

Answer 6. (*Continue*)