

MTH6016 Group Theory – Coursework 1

October 4, 2023

This coursework counts for 4% of your mark for this module. You should answer all questions, and each question will be marked out of 4. You should give full explanation of your answers. Please submit your solutions on QMPlus by 2pm on Friday 13th October. Your submission must be entirely your own work.

1. Suppose that G is a group. Let $f, g \in G$ and suppose that $\text{ord}(f) = 3$ and $\text{ord}(g) = 6$.
 - (i) What is $\text{ord}(f^2)$?
 - (ii) What is $\text{ord}(g^3)$?
 - (iii) If $fg = gf$, show that the order of fg is a factor of 6.
2. In \mathcal{U}_{31} , find all elements of the subgroup $\langle 4, 30 \rangle$.
3. Write down a Cayley table for \mathcal{U}_9 and state the order of each element.
4. List all of the elements of $\text{GL}_2(\mathbb{F}_2)$ and find the order of each element.
5. Find two elements g, h of $\text{GL}_2(\mathbb{F}_2)$ such that $\langle g, h \rangle = \text{GL}_2(\mathbb{F}_2)$.
6. Let X be any set and let $\mathcal{P}(X)$ denote the set of all subsets of X . Given any two subsets $A, B \subseteq X$, let $A \triangle B$ denote their symmetric difference $A \triangle B = (A \cup B) \setminus (A \cap B)$, which is the set of all elements of X which belong to *exactly one* of A and B . Show that $\mathcal{P}(X)$ is a group with respect to the binary operation \triangle .