Powers and Logarithms

Dr Lubna Shaheen

SEF041 - Mathematics B

Queen Mary University of London

Table of Contents

How Logarithms are useful

Biology	Calculate Growth of bacteria or viruses using Log Machine	*Some bacteria grow double after 20 minutes, to check how much will it grow in 24 hours, biologist use log. Log is easy to interpret on calculator*
Archaeologist	Use logs and exponents to determine how old a fossil or artifact is	
Engineering	To calculate the magnitude of Earthquake	*The magnitude of an earthquakes is measured from the log of an amplitude of waves recorded by seismographs.*
Physics, Chemistry, Philosophy	Log used anywhere to manage long calculations	*Main purpose is to simplify the long calculations, we usually encounter long numbers which are not easy to divide or multiply, so we use log to short those numbers and then divide or multiply and then use ani-log to get the real numbers.*

Powers and Logarithms

What are the exponents: Exponents are repeated multiplication.

Exponents invlove two numbers

Exponents are higher integer values

Powers and Logarithms

Exponential Law

1. $x^1 = x$
2. $x^m x^n = x^{m+n}$
3. $x^0 = 1$
4. $\dfrac{x^m}{x^n} = x^{m-n}$

Powers and Logarithms

Exponential Law

1. $x^{-n} = \dfrac{1}{x^n}$
2. $(xy)^m = x^m y^m$
3. $(x^m)^n = x^{mn}$
4. $(\dfrac{x}{y})^n = \dfrac{x^n}{y^n}$

Powers and Logarithms

Example

- What is $1^1 =?$, $1^{100=?}$, $1^{-5} =?$
- Write $5p^2q^3 \times 3pq^4$ in its simplest form.
- Express $\frac{1}{3^{-4x}}$ in the form of a^x for a suitable number a.

Example: Simplify each expression given below

(i) $\sqrt[3]{8}$

(ii) $\sqrt[3]{-125}$

(iii) $\sqrt[3]{40}$

(iv) $\left(2\dfrac{3}{4}\right)^5$?

Powers and Logarithms

Exercise:

(i) 2^6

(ii) $8^{2/3}$

(iii) $\sqrt[3]{1000000}$

(iv) $(0.2)^{-2}$

Common Mistakes:

$$x^m + x^n \neq x^{m+n}$$

$$(x + y)^n \neq x^n + y^n$$

$$(x^m)^n \neq x^{m^n}$$

Powers and Logarithms

Simplify: $$\frac{\sqrt[7]{a^{21}}}{(a^2a^3\sqrt{a})^4}$$

Simplify: $\sqrt[3]{8}$

Powers and Logarithms

Exponentials: For exponentials we are interested where the base is fixed and the exponent is the variable. Fortunately calculations with exponentials follow the rules of powers as discussed above. $y = a^x$

Example

Solve the equations

(i) $10^{1-x} = 10^4$ (ii) $4^{5-9x} = \frac{1}{8^{x-2}}$

(iii) $10^{2x} = 10000$,

Powers and Logarithms

Powers and Logarithms

Common Equations

Exponential Growth or Decay: $y(t) = ae^{kt}$
Where y(t)= value at time "t", a = value at the start, k= rate of growth (when $k > 0$) or decay (when $k < 0$), t=time.

Newton's law of cooling:

$$T(t) = Ae^{kt} + T_s$$

Logistic Growth:

$$f(x) = \frac{c}{1 + ae^{-bx}}$$

Powers and Logarithms

Definition: Logarithms are a way of looking differenty at exponents or indices or powers. In its simplest form, a logarithm answers the followings questions: **How many of one number do we multiply to get another number?**

Example: Consider the expression $\mathbf{2^5}$. This is an abbreviation of

$$\mathbf{2} \times \mathbf{2} \times \mathbf{2} \times \mathbf{2} \times \mathbf{2} \times \mathbf{2}$$

it seems its doubling something and keeps growing. This will make the equation $\mathbf{2^5} = \mathbf{32}$. **Now there are 3 no's we discuss**

(i) The number we multiply is called the "base/growth raet" and

(ii) "5 means how many times you will grow at that rate"

(iii) Now what is 32? Its the final result, it means you are 32 times bigger than you started.

Powers and Logarithms

Now we will see how this analogy of time and growing will help us:

Example As we seen already $2^5 = 32$, now lets see the following

$$2^0 = 1 \text{ (using index law)}$$

You have not grown at all, you are of same size as you started

$$2^{-1} = \frac{1}{2} \text{ (using index law)}$$

How to write it: $2^5 = 32 \iff \log_2(32) = 5$, we read this mathematical term as log base 2 of 32 equalls to 5.

$$2^0 = 1 \iff \log_2(1) = 0$$

$$2^{-1} = \frac{1}{2} \iff \log_2(\frac{1}{2}) = -1, \text{ yes lograthims can be negative numbers}$$

Powers and Logarithms

Example

What is $\log_3(81)$......?
I am no more at doubling machine, instead I am trippling , we are asking "how many 3s need to be multiplied together to get 81?"

$$3 \times 3 \times 3 \times 3 = 81, \text{ so we need 4 of the 3s}$$

Answer: $\log_3(81) = 4$.

Example: $\log_{\sqrt{3}}(27) = ?$

The question is, what power I must raise to get 27.
Lets try $\sqrt{3} \times \sqrt{3} \times \sqrt{3} \times \sqrt{3} = 3 \times 3 = 9$, this gives us 9, how many times do we need to get 27 this means we need $\sqrt{3}$, 2 more times,

$$\sqrt{3} \times \sqrt{3} \times \sqrt{3} \times \sqrt{3} \times \sqrt{3} \times \sqrt{3} = 27$$

How to write this in index form: Fractional indices

$$\left(3^{\frac{1}{2}}\right)^6 = 3^3 = 27$$

Powers and Logarithms

Practice Questions

(i) $\log_5 25 =$

(ii) $\log_3 81 =$

(ii) $\log_2 32 =$

(iv) $\log_{10} 1000 =$

(v) $\log_4 1 =$

(vi) $\log_4(4) =$

(vii) $\log_2(\frac{1}{2} =$

(viii) $\log_3(\frac{1}{27}) =$

(ix) $\log_2(\frac{1}{16}) =$

(x) $\log_a(a^3) =$

(xi) $\log_4(-1) =$

Powers and Logarithms

Answers

$$\log_5 25 = \mathbf{2}$$

$$\log_3 81 = \mathbf{4}$$

$$\log_2 32 = \mathbf{5}$$

$$\log_{10} 1000 = \mathbf{3}$$

$$\log_4 1 = \mathbf{0}$$

$$\log_4 4 = \mathbf{1}$$

$$\log_2\left(\frac{1}{2}\right) = \mathbf{-1}$$

$$\log_3\left(\frac{1}{27}\right) = \mathbf{-3}$$

$$\log_2\left(\frac{1}{16}\right) = \mathbf{-4}$$

$$\log_a(a^3) = \mathbf{3}$$

$$\log_4(-1) = \text{NOPE}$$

Powers and Logarithms

Logarithms Rules

1. Identity Rule: $\log_a(a) = 1, \quad \log_a(1) = 0$
2. Product Rule: $\log_a(xy) = \log_a(x) + \log_a(y)$
3. Quotient Rule: $\log_a(\frac{x}{y}) = \log_a(x) - \log_a(y)$
4. Power Rule: $\log_a(x^n) = n \log_a(x)$
5. Change of Base Rule: $\log_a(x) = \dfrac{\log_b x}{\log_b a}$
6. Equality Rule: If $\log_a x = \log_a y$ then $x = y$.

Powers and Logarithms

Addition and Subtraction Rule

Example

(i) $5\log_8(2) + \frac{1}{2}\log_8(4)$

(ii) $\log_3(270) - (\log_3(2) + \log_3(5))$

(iii) $\log_a(a^2) + 3\log_a(a)$

Powers and Logarithms

Logarithms to Exponentials and Exponentials to Logarithms

The equation $a^x = c$ is equivalent to the equation $x = \log_a(c)$

Example

(i) $7^x = 49$

(ii) $3^x = 81$

(iii) $10^x = 10000$

(iv) $5^x = 0.2$

(v) $2^x = -8$

Powers and Logarithms

Rule of change of base for logarithms

$$\log_b(c) = \frac{\log_a c}{\log_a b}$$

Example

Solve $4^x = 64$ using base $a = 4$ and $a = e$.

Powers and Logarithms

Powers and Logarithms

Solving equations with exponentials and logarithms

Example

Given that $\ln a = 2$ solve $a^x = e^6$ for x.

Powers and Logarithms

Practice Question

Solve the simultaneous equation:

$$\log_2 y = \log_2 x + 4$$
$$8^y = 4^{2x+3}$$

Disguised Quadratics

Example

Solve $e^{2x} + e^{x} = 2$ for x.

Powers and Logarithms

Practice Questions

1. $y^4 - 5y^2 - 36 = 0$
2. $q - 5\sqrt{a} - 36 = 0$
3. $2^{2x} - 2^{x+1} + 1 = 0$
4. $9(1 + 9^{x-1}) = 10 \times 3^x$

Powers and Logarithms

Homework Worksheet 1

Exams Style Questions

Question 1: Solve for x and y the following system of simultaneous equations:

$$2\ln(x) = \ln(y) + \ln(5)$$

$$e^x e^y = e^3$$

Question 2: Find an expression for x given that $\ln(a) = 2$ and $a^x = e^6$.

Homework Worksheet 1

Exams Style Questions

Homework Worksheet 1

Exams Style Questions

Question 3: Find the values of x satisfying

$$\log_2(x) + \log_2(x+5) = 2$$

Question 4 (2022): Solve the following logarithmic equation:

$$16\log_2 x + 4\log_4 x + 2\log_{16} x = 37, \quad x > 0.$$

Question 5 (2023): $2\log\left(\frac{x}{y}\right) - 1 = \log(10x^2y), \quad x \neq 0, y \neq 0$
Find the exact value of y.

Homework Worksheet 1

Exams Style Questions

Homework Worksheet 1

Exams Style Questions

Question 6: Why Can't Logarithms Argument Be Negative? Give Reason.

Question 7 (2023): Given that $a > 0, b > 0$ and $y > 0$, and that

$$2 + \log_a b + 3\log_a y = 2\log_a(a^2 y)$$

express y in terms of a and b, in form **not** involving logarithms.

Homework Worksheet 1

Exams Style Questions

Activities for the Week 1

- Complete the Tasks of Week 1
- Attend your Lecture on Monday or Thursday
- Attempt End of Week 1 Quiz, Tutorial Sheet 1, sort out problems in Homework Worksheet 1
- Discuss any concerns at the Week 2 Feedback and support session on Friday or during the upcoming Lectures and Tutorials.
- Start Week 2 Tasks

End of Lecture