Science and Engineering Foundation Programme
Assessment 1

SEF041& MED3001 - Mathematics B

Tuesday 5 November 2019, 11.00am

Time Allowed: 50 minutes
Note:
The mark will be calculated from **the best ANSWERS to FOUR** questions out of **FIVE**.
All questions carry equal marks.

FULL NAME:

STUDENT ID:

TUTOR GROUP:

Instructions:

DO NOT TURN OVER THE SCRIPT until the test has started.

Make sure you read all questions carefully.

CALCULATORS:
Only Casio fx-82, fx-83 and fx-85 are allowed (+ extensions like ES, GT, GT plus).

For marking purposes only:

Question	Marks	Comments
1		
2		
3		
4		
5		
TOTAL :		

1. (a) Rewrite as a power of a:

$$\sqrt[3]{a}.$$

[2]

(b) Simplify as far as possible:

$$\frac{\sqrt[3]{a^7\sqrt{a}}}{a^{-4}}.$$

[8]

(c) Change to base 2 and simplify as far as possible:

$$\log_4 \sqrt{x}.$$

[5]

(d) Solve for x

$$1 + \log_2(x^2) + \log_4 \sqrt{x} = 4.$$

Give your solution in its simplest surd (exact) form. **[10]**

Write your solutions here

a) $\sqrt[3]{a} = a^{\frac{1}{3}}$

b)

$$\frac{\sqrt[3]{a^7\sqrt{a}}}{a^{-4}} = \frac{\sqrt[3]{a^7 \cdot a^{1/2}}}{a^{-4}} =$$

$$= \frac{\sqrt[3]{a^{15/2}}}{a^{-4}} = \frac{\left(a^{15/2}\right)^{1/3}}{a^{-4}} =$$

$$= \frac{a^{5/2}}{a^{-4}} = a^{\frac{5}{2}+4} = a^{13/2}$$

c) $\log_4 \sqrt{x} = \frac{\log_2 x^{\frac{1}{2}}}{\log_2 4} = \frac{\frac{1}{2}\log_2 x}{2} = \frac{1}{4}\log_2 x$

d)

$$1\ \log_2 x^2 + \log_4 \sqrt{x} = 3$$

$$2\log_2 x + \frac{\log_2 \sqrt{x}}{\log_2 4} = 3$$

$$2\log_2 x + \frac{1}{2}\log_2 x^{1/2} = 3$$

$$2\log_2 x + \frac{1}{4}\log_2 x = 3$$

$$\frac{9}{4}\log_2 x = 3$$

$$\log_2 x = \frac{3}{9/4} = \frac{4}{3}$$

$$x = 2^{4/3} = \sqrt[3]{2^4} = \sqrt[3]{16}$$

2. (a) The quadratic is specified as

$$f(x) = x^2 + 4x + k$$

i) Find values of k for which the equation $f(x) = 0$ has a repeated root. **[5]**

ii) For $k = 3$ write $f(x)$ in a factorised form. **[5]**

iii) For $k = 2$ what is the minimum value of $f(x)$ and what is its symmetry axis. **[8]**

(b) When the polynomial $g(x) = x^3 + x^2 - px - 2$ is divided by $x - 3$, the remainder is 1. Find the value of the constant p. **[7]**

Write your solutions here

a)

i) $\Delta = 0$ repeated root.

$$\Delta = 4^2 - 4k = 16 - 4k = 0$$

$$k = 4$$

ii)

$$f(x) = x^2 + 4x + 3 = (x + 1)(x + 3)$$

iii)

$$f(x) = x^2 + 4x + 2 = (x + 2)^2 - 4 + 2 = (x + 2)^2 - 2$$

So the symmetry axis is at $x = -2$ and the minimum is $y = -2$.

Alternatively:

Symmetry axis is at

$$x = -\frac{b}{2a} = -\frac{4}{2} = -2$$

and the minimum is at

$$f(-\frac{b}{2a}) = f(-2) = (-2)^2 + 4(-2) + 2 = 4 - 8 + 2 = -2$$

b)

b)

$$\frac{x^3 + x^2 - px - 2}{x - 3} \rightarrow r = 1$$

$$f(3) = 1$$

$$f(3) = 3^3 + 3^2 - 3p - 2 = 1$$

$$\Rightarrow 27 + 9 - 2 - 3p = 1$$

$$34 - 3p = 1$$

$$3p = 33$$

$$p = 11$$

3. Consider the three points $A = (0, -1)$, $B = (2, 3)$ and $C = (5, 1)$.

(a) Find the equation of the line segment AB in an implicit form. **[5]**

(b) Find the shortest distance of point C from the line segment AB. **[5]**

(c) Find the equation of the perpendicular bisector of the line segment AB. **[15]**

Write your solutions here

a)

$A = (0,-1)$, $B = (2,3)$, $C = (5,1)$

i)

AB line

$\text{grad } AB = \frac{\Delta y}{\Delta x} = \frac{3+1}{2-0} = \frac{4}{2} = 2$

$y = 2x + c$

going through $A = (0,-1)$

$-1 = 0 + c \rightarrow c = -1$

$y = 2x - 1$ ← line AB

$2x - y - 1 = 0$ ← implicit form

$a = 2,\ b = -1,\ c = -1$

ii)

b)

Shortest distance is the perpendicular distance:

$a = 2,\ b = -1,\ c = -1$

ii)

$C = (5,1)$ $\quad d_c = \frac{ax_c + by_c + c}{\sqrt{a^2+b^2}}$

$d = \left| \frac{2 \cdot 5 + (-1) \cdot 1 - 1}{\sqrt{2^2 + (-1)^2}} \right| =$

$= \left| \frac{10-2}{\sqrt{4+1}} \right| = \frac{8}{\sqrt{5}} = \frac{8\sqrt{5}}{5}$

c) Midpoint AB = (1, 1).
Perp.gradient = $-\frac{1}{2}$.
So

$$y = -\frac{1}{2}x + c$$

We use the midpoint:

$$1 = -\frac{1}{2} + c$$

$$c = \frac{3}{2}$$

The equation of perp. bisector is:

$$y = -\frac{1}{2}x + \frac{3}{2}$$

4. (a) Consider the following equation of a circle:

$$(x-1)^2 + \left(y - \frac{2}{3}\right)^2 = \frac{4}{9}$$

i) Specify the coordinates of the centre. **[2]**

ii) Specify the radius. **[2]**

iii) Write it in a parametric form. **[6]**

(b) Let $A = (1, 2)$ and $B = (1, 1)$. Derive an equation specifying the locus of all points $P = (x, y)$ such that the distance $\overline{AP}$ is 2 times as long as the distance $\overline{BP}$. Clearly specify which curve you obtained. **[15]**

Write your solutions here

a)

i) Centre = $(1, \frac{2}{3})$

ii) Radius = $\frac{2}{3}$

iii) Parametric form:

$$x = 1 + \frac{2}{3}\cos\theta$$

$$y = \frac{2}{3} + \frac{2}{3}\sin\theta$$

b)

$\sqrt{(x-1)^2 + (y-2)^2} = 2\sqrt{(x-1)^2 + 4(y-1)^2}$

$(x-1)^2 + (y-2)^2 = 4(x-1)^2 + 4(y-1)^2$

$(x-1)^2 + (y-2)^2 = 4(x-1)^2 + 4(y-1)^2$

$x^2 - 2x + y^2 - 4y + 5 - 4x^2 + 8x - 4y^2 + 8y - 8 =$

$3x^2 - 6x + 3y^2 - 4y + 3 = 0$

$x^2 - 2x + y^2 - \frac{4}{3}y + 1 = 0$

$(x-1)^2 + (y - \frac{2}{3})^2 - 1 - \frac{4}{9} + 1 = 0$

$(x-1)^2 + (y - \frac{2}{3})^2 = \frac{4}{9}$

The curve is the circle with the Centre = $(1, \frac{2}{3})$ and radius = $\frac{2}{3}$.

5. Function $g(x)$ is specified as

$$g(x) = x^2 - 3.$$

(a) Specify the domain and range for $g(x)$. **[2]**

(b) What is the domain and range so that $g(x)$ becomes invertible? **[3]**

(c) Find the inverse function $g^{-1}(x)$ and specify the domain and range of the inverse function. **[5]**

(d) Specify if function $g(x)$ is odd, even or neither. **[7]**

(e) Compose $g(x)$ with $f(x) = e^x$, i.e. find: $(f \circ g)(x) = f(g(x))$ and $(g \circ f)(x) = g(f(x))$. **[8]**

Write your solutions here

(a) Domain $D = \mathbb{R}$
Range $R = [-3, \infty)$

(b) Domain and range so that $g(x)$ becomes invertible: $D = [0, \infty)$ and $R = [-3, \infty)$.
(Or equivalently $D = (-\infty, 0]$ and $R = [-3, \infty)$)

(c) Inverse function $g^{-1}(x)$:

$$y = x^2 - 3.$$
$$x = y^2 - 3$$
$$y^2 = x + 3$$
$$y = \pm\sqrt{x+3}$$

For $D = [0, \infty)$ and $R = [-3, \infty)$ we take $g^{-1}(x) = \sqrt{x+3}$ whose domain is $D = [-3, \infty)$ and range $R = [0, \infty)$.

(Or equivalently, if $D = (-\infty, 0]$ and $R = [-3, \infty)$ then $g^{-1}(x) = -\sqrt{x+3}$ whose domain is $D = [-3, \infty)$ and range $R = (-\infty, 0]$)

(d) $g(-x) = (-x)^2 - 3 = x^2 - 3 = g(x)$ is even.

(e)

$$(f \circ g)(x) = f(g(x)) = e^{x^2-3}$$

and

$$(g \circ f)(x) = g(f(x)) = e^{2x} - 3$$

End of Assessment