Solutions to M1 Mid term 2016

1.a) $\frac{a^4 a^{-2}}{(a^{-1})^3}\sqrt{\frac{1}{a^3}} = a^5\sqrt{\frac{1}{a^3}} = a^5\sqrt{a^{-3}}$

$\frac{a^4 a^{-2}}{(a^{-1})^3}\sqrt{\frac{1}{a^3}} = a^5 a^{-\frac{3}{2}} = a^{\frac{10-3}{2}} = a^{\frac{7}{2}}$

b)

$2 \times 3^x = 1 - 3^{2x}$,

$2u - 1 + u^2 = 0$, Solutions of the quadratic eq. are

$u_1 = \sqrt{2} - 1, u_2 = -\sqrt{2} - 1$;

So only positive root for u can be used to solve for x

$3^x = \sqrt{2} - 1 = 0.414\,21 \Rightarrow x = \log_3\left(\sqrt{2} - 1\right) = -0.802\,26$ = - 0.802 (3d.p.)

EXACT rounded to 3dp

2. $\frac{x^3+4x^2-7x+6}{x+4} = x^2 - 7 + \frac{34}{x+4}$

$(x+4)\,|x^3 + 4x^2 - 7x + 6$

$x^2 \times \quad x^3 + 4x^2$

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$0 + 0 \quad -7x + 6$

$-7 \times \quad -7x - 28$

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$0 + 34$

3. Perp. bisector of $P(-2,1)$ and $Q(4,3)$

grad$PQ = \frac{3-1}{4+2} = \frac{2}{6} = \frac{1}{3} = m$

$m_\perp = -3$

Midpoint $PQ\,(1,2)$

perp.bis: $\frac{y-2}{x-1} = -3 \Rightarrow y - 2 = -3(x-1) = -3x + 3$

$y = -3x + 5$

b) $P(-2,1)$ the centre

$(x+2)^2 + (y-1)^2 = r^2$

going through Q:

$(4+2)^2 + (3-1)^2 = 6^2 + 2^2 = 36 + 4 = 40 = r^2 \Rightarrow r = \sqrt{40}$

$(x+2)^2 + (y-1)^2 = 40$

in polar coordinates:

$x = -2 \pm \sqrt{40}\cos\theta$

$y = 1 \pm \sqrt{40}\sin\theta$

4. $f(x) = x^2 - 4x + 7$ a) Completing the square:

$$f(x) = x^2 - 4x + 7 = (x-2)^2 - 4 + 7 = (x-2)^2 + 3$$

b) For determining the domain and range so that $f(x)$ is invertible it is good to notice that $x \to x - 2$ i.e. x^2 was translated to the right by 2 units.

So for the domain we need to choose only one branch, either: $D = [2, +\infty) = \{x \geq 2\}$

(or $D = (-\infty, 2] = \{x \leq 2\}$) so that the function is $1 - 1$.

For the range, again we notice that $y \to y - 3$ so the x^2 was translated upwards by 3 units.

For the range we need to choose: $R = [3, \infty)$ so that the function is onto.

c) $f : [2, +\infty) \to [3, \infty)$ the inverse:

$y = (x-2)^2 + 3$

replacing x with y gives: $x = (y-2)^2 + 3$.

Now isolating y:

$x - 3 = (y-2)^2 \to y - 2 = \pm\sqrt{x-3}$

So $y = \pm\sqrt{x-3} + 2$.

But $f : [2, +\infty) \to [3, +\infty)$ so $f^{-1} : [3, +\infty) \to [2, +\infty)$ Therefore we need to choose $+$ and $f^{-1}(x) = \sqrt{x-3} + 2$.

d)

$g(x) = \frac{4x}{x^2+2}$

$$g(-x) = \frac{-4x}{(-x)^2 + 2} = -\frac{4x}{x^2+2} = -g(x)$$

So $g(x)$ is odd.