MTH6107 Chaos & Fractals

Exercises 1

(A) Suppose the map $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = x^2 - 6x + 10$.

Exercise 1. Draw the graph of the map f, and determine all its fixed points. Determine which of these points are attracting and which of these points are repelling.

Exercise 2. For the map f, determine an eventually fixed point which is not a fixed point.

Exercise 3. Draw a graph of the map f^2. Determine all the points of prime period 2 of f. Determine which of these points are attracting and which of these points are repelling.

(B) Now suppose the map $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = x^2 - 7/4$.

Exercise 4. Draw the graph of the map f, and determine all its fixed points. Determine which of these points are attracting and which of these points are repelling.

Exercise 5. For the map f, determine an eventually fixed point which is not a fixed point.

Exercise 6. Draw a graph of the map f^2. Determine all the points of prime period 2 of f. Determine which of these points are attracting and which of these points are repelling.

(C) Now suppose the map $f : \mathbb{R} \to \mathbb{R}$ is defined by

$$f(x) = \begin{cases} x + 1/2 & \text{for } x < 0 \\ -2x + 1/2 & \text{for } x \geq 0\,. \end{cases}$$

Exercise 7. Draw the graph of the map f, and determine all its fixed points. Determine which of these points are attracting and which of these points are repelling.

Exercise 8. For the map f, determine an eventually fixed point which is not a fixed point.

Exercise 9. Draw a graph of the map f^2. Determine all the points of prime period 2 of f. Determine which of these points are attracting and which of these points are repelling.

Exercise 10. For the map f, determine all its points of prime period 3.

Exercise 11. For the map f, determine all its points of prime period 4.

Exercise 12. Is it the case that f has a point of prime period n for every $n \in \mathbb{N}$?

Exercise 13. Can you guess (or even prove) a formula for the number of points of period n for the map f?