(1) Consider the curve

$$C = \{(\sin t,\ t) \in \mathbb{R}^2 \mid 0 < t < 2\pi\},$$

as well as the following parametrisation of C:

$$\gamma : (0, 2\pi) \to \mathbb{R}^2, \qquad \gamma(t) = (\sin t,\ t).$$

(b) [13 marks] Let C also be given the *downward* (decreasing y-value) *orientation*, and let $\mathbf{F}$ be the vector field on $\mathbb{R}^2$ given by

$$\mathbf{F}(x, y) = (-2,\ y^3).$$

Compute the following curve integral:

$$\int_C \mathbf{F} \cdot d\mathbf{s}.$$

Bad answer:

$\gamma\ (\sin t, t) \qquad \gamma'\ (\cos t, 1)$

$F\ (-2, t^3)$

There are correct things here, but please write correct statements in your work!

(e.g. γ'(t) = (cos t, 1))

$\int_C F \cdot ds$

$(-2, t^3) \cdot (\cos t, 1)$

$-2\cos t + t^3 \cdot 1$

$= -2\cos t + t^3$

$-\int_0^{2\pi} (-2\cos t + t^3)\, dt$

$-(-2\sin t + \frac{1}{4} t^4)$

$2\sin t - \frac{1}{4} t^4 \Big|_0^{2\pi}$

$0 \quad -\frac{1}{4}(2\pi)^4$

$-4\pi^4$

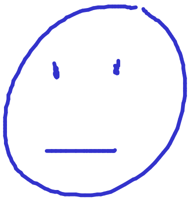
Again, many correct things in this mess, but this is completely unreadable.
No one can follow the steps.

What is equal to what?
What is the sequence of steps from start to end?

If you make me try to figure all this out, then I will be very unhappy with you. :(

(You will lose marks for your answer being gibberish.)

OK answer:



$\gamma(t) = (\sin t, t)$ $\qquad$ $\gamma'(t) = (\cos t, 1)$

$F(\gamma(t)) = (-2, t^3)$ (sin t, t)

computations are fine

*** Why did you compute this integral using γ? Why can you do this in the first place? ***

$$\int_C F\cdot ds = -\int_0^{2\pi} F(\gamma(t))\cdot \gamma'(t)_{\gamma(t)}\, dt$$

Why the "-" sign here?

$$= -\int_0^{2\pi} (-2, t^3)\cdot(\cos t, 1)\, dt$$

$$= -\int_0^{2\pi} (-2\cos t + t^3)\, dt$$

$$= -\left(-2\sin t + \tfrac{1}{4}t^4\right)\Big|_0^{2\pi}$$

$$= -\tfrac{1}{4}(2\pi)^4$$

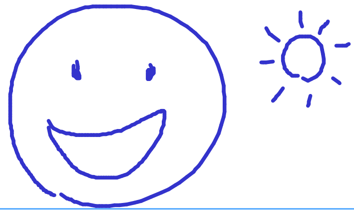
$$= \boxed{-4\pi^4}$$

This part is very good.

Here, the computations are very well done, but much of the explanations around them are missing. (You will lose a few marks for this.)

Show that you understand more than just number crunching!

Good answer:



γ - injective param. of C
- image of γ = all of C.

$\Rightarrow$ Can use γ to compute integral

γ generates upward orientation of C
(y-component increasing)
$\Rightarrow$ opposite to orientation of C.

$\gamma(t) = (\sin t, t)$
$\gamma'(t) = (\cos t, 1)$
$F(\gamma(t)) = (-2, t^3)_{(\sin t, t)}$

$$\Rightarrow \int_C F \cdot ds = -\int_0^{2\pi} F(\gamma(t)) \cdot \gamma'(t)_{\gamma(t)}\, dt$$

$$= -\int_0^{2\pi} (-2, t^3) \cdot (\cos t, 1)\, dt$$

$$= -\int_0^{2\pi} (-2\cos t + t^3)\, dt$$

$$= -\left(-2\sin t + \tfrac{1}{4}t^4\right)\Big|_0^{2\pi}$$

$$= -\tfrac{1}{4}(2\pi)^4$$

$$= \boxed{-4\pi^4}$$

This is similar to the previous answer, except it also shows the reasoning for how the curve integral is converted to a calculus integral.

Note that you don't have to write much to have a good answer! You just have to show understanding of every step in the process.