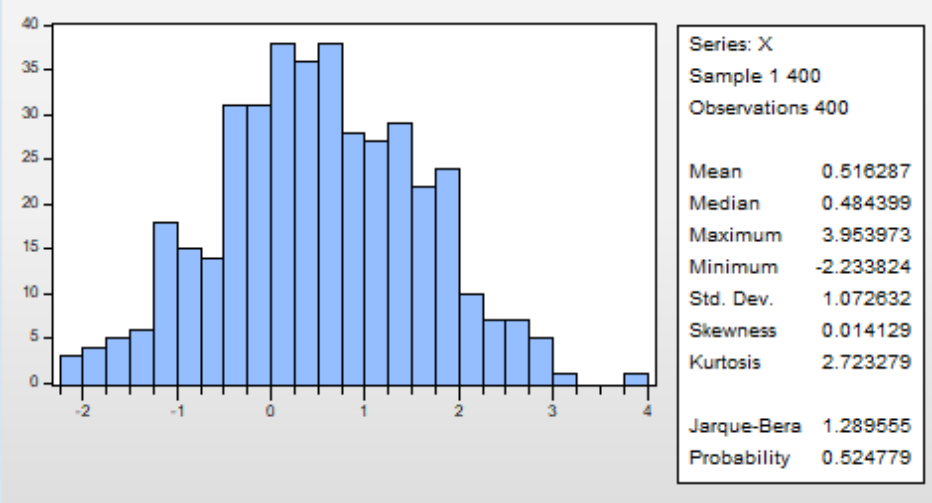
Solution Quiz 3

Question 3.

The researcher computed summary statistics of the time series X using a sample containing $N = 100$ observations, see the output below.

1. Test whether skewness $S(X)$ equals 0.
2. Test whether kurtosis $K(X)$ equals 3
3. Test whether X has normal distribution



Solution of Question 3.

1. We test the hypothesis

H_0: $S(X) = 0$ against alternative H_1: $S(X) \neq 0$.

at 5% significance level.

We construct the test statistics:

$$t = \frac{\hat{S}(X)}{\sqrt{6/N}} = \frac{0.014129}{\sqrt{6/400}} = 0.11536$$

Under the null hypothesis, $t \sim N(0,1)$ is normally distributed.

Rule: we reject H_0 at 5% significance level, if

$$|t| \geq 2.$$

In our case $|t| = 0.11536 < 2$. Hence, the test shows that there no evidence in the data to reject the null hypothesis of zero skewness $S(X) = 0$.

2. Next we test at 5% significance level the hypothesis:

H_0: $K(X) - 3 = 0$ against alternative H_1: $K(X) \neq 3$.

We use the test statistics:

$$t = \frac{\hat{K}(X) - 3}{\sqrt{24/N}} = \frac{2.723 - 3}{\sqrt{24/400}} = -1.1308.$$

By theory, under null hypothesis, $t \sim N(0,1)$ is normally distributed. Therefore the testing rule is similar as for testing skewness:

Rule: reject H_0 at 5% significance level, if

$$|t| \geq 2.$$

In our case $|t| = 1.1308 < 2$. Hence, the test does not rejects the null hypothesis of zero kurtosis $K(X) = 3$.

3. Jargue-Bera test is used to test the hypothesis:

H_0: $S(X) = 0$ and $K(X) - 3 = 0$ ("normal distribution")

against alternative

H_1: $S(X) \neq 0$ or $K(X) - 3 \neq 0$ ("distribution is not normal").

Normal distribution has $S(X) = 0$ and $K(X) = 3$. Thus, in case of normal distribution, test will not reject H_0.

Test will reject H_0 if either skewness is not 0 or if kurtosis is not 3. That will indicate that distribution is not normal.

Since the $p = 0.5247$ value of Jargue-Bera test is larger than 0.05 we do not reject the asymptotic normality at significance level 5%.