Solution Quiz 3

Question 1. a) Suppose that

$$X_t = \varepsilon_t + t, \quad t = 1, 2, ...$$

where ε_t is a white noise sequence with zero mean and variance $E\varepsilon_t^2 = 1$. Investigate whether time series X_t is covariance stationary.

b) Suppose that

$$X_t = t\varepsilon_t, \quad t = 1, 2, ...$$

where ε_t is a white noise sequence with zero mean and variance $E\varepsilon_t^2 = 1$. Investigate whether time series X_t is covariance stationary.

Solution of Question 1.

Time series X_t is a covariance stationary time series if it satisfies three properties:

– $EX_t = \mu$ for all t (does not depend on t);

– $Var(X_t) = \sigma^2$ for all t (does not depend on t);

– $Cov(X_t, X_{t-k}) = \gamma_k$ - covariance function depends only on the lag k and does not depend on t.

a) We have

$$EX_t = E[\varepsilon_t + t] = E[\varepsilon_t] + t = 0$$

since by assumption $E[\varepsilon_t] = 0$. We see that mean EX_t changes with t. Therefore, this time series is not covariance stationary.

b) We have

$$EX_t = E[t\varepsilon_t] = tE[\varepsilon_t] = 0$$

since by assumption $E[\varepsilon_t] = 0$. Hence, mean EX_t does not change with t.

Next we compute the variance:

$$\text{Var}(X_t) = E(X_t - EX_t)^2 = EX_t^2 = E[(t\varepsilon_t)^2] = t^2E[\varepsilon_t^2] = t^2.$$

since by assumption $E[\varepsilon_t^2] = 1$. We see that the variance $\text{Var}(x_t)$ changes with t. Therefore, this time series is not covariance stationary.

Question 2. Explain why the following sequence

$$\rho_1 = 0.8, \quad \rho_2 = 0.5, \quad \rho_3 = \rho_1 + \rho_2, \quad \rho_4 = \rho_1 + \rho_2 + \rho_3, ...$$

cannot be the auto-correlation function of a covariance stationary sequence.

Solution of Question 2.

Correlation function ρ_k at lag k of a covariance stationary time series X_t satisfies three properties:

– $\rho_0 = 1$
– $\rho_k = \rho_{-k}$ for any lag $k = 1, 2, 3, ...$
– $|\rho_k| \leq 1$ for any k.

We have that

$$\rho_3 = \rho_1 + \rho_2 = 0.8 + 0.5 = 1.3 > 1.$$

Therefore ρ_k cannot ne correlation function of a covariance stationary time series.

Question 3. Using the following EVIEWS correlogram of time series Xt, determine whether x_t is a white noise time series (series of uncorrelated random variables).

Correlogram of Y

Date: 29/11/20 Time: 10:53
Sample: 1 400
Included observations: 400

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.023	-0.023	0.2196	0.639
		2	-0.032	-0.033	0.6458	0.724
		3	-0.036	-0.037	1.1600	0.763
		4	-0.057	-0.060	2.4792	0.648
		5	0.050	0.045	3.4877	0.625
		6	-0.076	-0.080	5.8704	0.438
		7	0.067	0.063	7.7087	0.359
		8	-0.009	-0.012	7.7392	0.459
		9	0.012	0.016	7.7940	0.555
		10	-0.041	-0.049	8.4897	0.581

Brief solution of question 3.

(i) (**Testing for correlation using ACF**.) Time series is a white noise if it is serially uncorrelated, that is $\rho_k = 0$ for $k \geq 1$. Hence, to test for white noise, we test the hypotheses

$H_0: \ \rho_k = 0$ against alternative $H_1: \ \rho_k \neq 0$
at each lag $k = 1, 2, ...$ at significance level 5%.

Rule: ACF ρ_k at lag k is significantly different from zero at 5% significance level if $|\hat{\rho}_k| > 2/\sqrt{N}$, where N is the number of observations.

If $|\hat{\rho}_k| \leq 2/\sqrt{N}$, then ACF at lag k is not significantly different from 0.

If time series is white noise, then we do not reject H_0 for any $k = 1, 2, ...$

(ii) **Ljung-Box test**. This test can be also used to test for zero correlation. We select $m = 1, 2, ...$ and test the hypothesis

$H_0: \ \rho_1 = ...\rho_m = 0$ against alternative

$H_1: \ \rho_j \neq 0$ for some $j = 1, ..., m$.

We reject the H_0 at 5% significance level, if $p-$ value satisfies $p < 0.05$. If time series is white noise, then we do not reject H_0 for any $m = 1, 2, ...$

(iii) We have $2/\sqrt{N} = 2/\sqrt{400} = 0.1$. Since $|\rho_1| = 0.023 < 0.1$, $|\rho_2| = 0.032 < 0.1$, ..., and so on. We find that ρ_k is not significant at any lag $k = 1, 2, ...$ at 5% significance level, because $|\rho_k| \leq 2/\sqrt{N} = 2/\sqrt{400} = 0.1$. So, this time series is a white noise.

From the correlogram we see that p-values of Ljung Box test satisfy $p > 0.05$ for all $m = 1, ...$ So H_0 is not rejected at any m and this time series is a white noise.