Solutions of ECOM073 TEST 2024

Question 1 (33 marks)

(i) Test for absence of correlation using the following Eview output.
Use all available information given in this output.
Justify and explain your answers.
The sample size is $N = 400$.

Correlogram of R

Date: 06/11/20 Time: 12:56
Sample: 1 400
Included observations: 400

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.032	0.032	0.4132	0.520
		2	0.071	0.070	2.4559	0.293
		3	0.034	0.030	2.9181	0.404
		4	0.072	0.065	5.0071	0.287
		5	-0.006	-0.014	5.0211	0.413
		6	0.056	0.046	6.2865	0.392
		7	0.007	0.001	6.3071	0.504
		8	-0.021	-0.032	6.4796	0.594
		9	0.033	0.032	6.9152	0.646
		10	0.013	0.008	6.9898	0.726

Solution of question 1.

(i) (**Testing for correlation using ACF**.) Time series is a white noise if it is serially uncorrelated, that is $\rho_k = 0$ for $k \geq 1$. Hence, to test for white noise, we test the hypotheses

$H_0: \ \rho_k = 0$ against alternative $H_1: \ \rho_k \neq 0$

at each lag $k = 1, 2, ...$ at significance level 5%.

Rule: ACF ρ_k at lag k is significantly different from zero at 5% significance level if $|\hat{\rho}_k| > 2/\sqrt{N}$, where N is the number of observations.

If $|\hat{\rho}_k| \leq 2/\sqrt{N}$, then ACF at lag k is not significantly different from 0.

(ii) **Ljung-Box test**. This test can be also used to test for zero correlation. We select $m = 1, 2, ...$ and test the hypothesis

$H_0: \ \rho_1 = ...\rho_m = 0$ against alternative

$H_1: \ \rho_j \neq 0$ for some $j = 1, ..., m$.

We reject the H_0 at 5% significance level, if $p-$ value satisfies $p < 0.05$. If time series is white noise, that we do not reject H_0 for any $m = 1, 2, ...$

(iii) We have $2/\sqrt{N} = 2/\sqrt{400} = 0.1$. Since $|\rho_1| = 0.023 < 0.1$, $|\rho_2| = 0.032 < 0.1$, ..., and so on. We find that ρ_k is not significant at any lag $k = 1, 2, ...$ at 5% significance level, because $|\rho_k| \leq 2/\sqrt{N} = 2/\sqrt{400} = 0.1$. So, this time series is a white noise.

From the correlogram we see that p-values of Ljung Box test satisfy $p > 0.05$ for all $m = 1, ...$ So H_0 is not rejected at any m and this time series is a white noise.

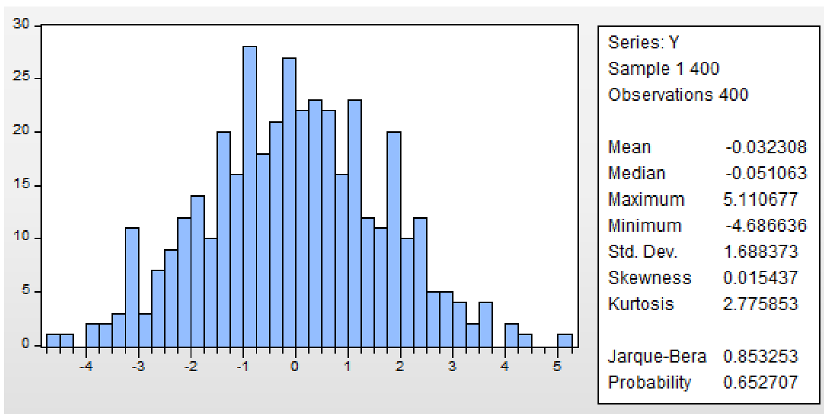
Question 2 (34 marks)

(i) Explain what is meant by a "stationary time series".

(ii) Test for significance of the skewness and excess kurtosis of returns r_t using summary statistics given below. Explain your testing procedures.

What can you say about the distribution of r_t?

The sample size is $N = 400$.



Solution of Question 2.

(i) Time series X_t is a covariance stationary time series if it satisfies three properties:

– $EX_t = \mu$ for all t (does not depend on t);

– $Var(X_t) = \sigma^2$ for all t (does not depend on t);

– $Cov(X_t, X_{t-k}) = \gamma_k$ - covariance function depends only on the lag k and does not depend on t.

In case of a non-stationary time series, one of these properties will be not satisfied, for example the mean EX_t could vary in time.

(ii) **Testing whether skewness $S(X) = 0$ (symmetry of distribution)**.

We test the hypothesis

H_0: $S(X) = 0$ against alternative H_1: $S(X) \neq 0$.

at 5% significance level.

We construct the test statistics:

$$t = \frac{\hat{S}(X)}{\sqrt{6/N}} = \frac{0.01543}{\sqrt{6/400}} = 0.12599$$

Under the null hypothesis, $t \sim N(0, 1)$ is normally distributed.

Rule: we reject H_0 at 5% significance level, if

$$|t| \geq 2.$$

In our case $|t| = 0.12599 < 2$. Hence, the test shows that there no evidence in the data to reject the null hypothesis of zero skewness $S(X) = 0$.

Testing whether kurtosis $K(x) = 3$ (non heavy tails).

Next we test at 5% significance level the hypothesis:

H_0: $K(X) - 3 = 0$ against alternative H_1: $K(X) \neq 3$.

We use the test statistics:

$$t = \frac{\hat{K}(X) - 3}{\sqrt{24/N}} = \frac{2.775 - 3}{\sqrt{24/400}} = -0.91856.$$

By theory, under null hypothesis, $t \sim N(0,1)$ is normally distributed. Therefore the testing rule is similar as for testing skewness:

Rule: reject H_0 at 5% significance level, if

$$|t| \geq 2.$$

In our case $|t| = 0.91856 < 2$. Hence, the test does not rejects the null hypothesis of zero kurtosis $K(X) = 3$.

Jargue-Bera test for asymptotic normality. Jargue-Bera test is used to test the hypothesis:

H_0: $S(X) = 0$ and $K(X) - 3 = 0$ ("normal distribution")

against alternative

H_1: $S(X) \neq 0$ or $K(X) - 3 \neq 0$ ("distribution is not normal").

Normal distribution has $S(X) = 0$ and $K(X) = 3$. Thus, in case of normal distribution, test will not reject H_0.

Test will reject H_0 if either skewness is not 0 or if kurtosis is not 3. That will indicate that distribution is not normal.

Since the $p = 0.6527$ value of Jargue-Bera test is larger than 0.05 we do not reject the asymptotic normality at significance level 5%.

Question 3 (33 marks)

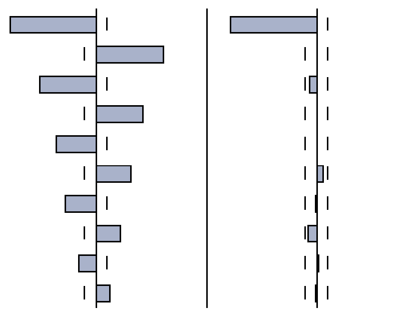
(i) Using the given Eviews output determine the order of AR(p) and MA(q) model you would fit to the data. Explain your answer.

The sample size is $N = 400$.

Correlogram of Y

Date: 04/10/20 Time: 09:19
Sample: 1 400
Included observations: 400

Autocorrelation | Partial Correlation



	AC	PAC	Q-Stat	Prob
1	-0.796	-0.796	255.30	0.000
2	0.637	0.010	419.30	0.000
3	-0.532	-0.061	534.03	0.000
4	0.445	0.005	614.45	0.000
5	-0.365	0.015	668.63	0.000
6	0.323	0.065	711.29	0.000
7	-0.290	-0.016	745.71	0.000
8	0.226	-0.080	766.62	0.000
9	-0.170	0.018	778.51	0.000
10	0.131	-0.003	785.60	0.000

Solution of Question 3:

(i) To select the order p for AR(p) model, we use the sample PACF function. We test the hypothesis

$H_0: \rho_k = 0$ against alternative $H_1: \rho_k \neq 0$

at lags $k = 1, 2, ...$ at significance level 5%, where ρ_k is the PACF function.

PACF $\hat{\rho}_k$ at lag k is significantly different from 0 at 5% significance level, if $|\hat{\rho}_k| > 2/\sqrt{N}$, where N is the number of observations.

If $|\hat{\rho}_k| \leq 2/\sqrt{N}$, then PACF at lag k is not significantly different from 0.

Rule: we select for p the largest lag k at which the PACF is significant.

This rule can be used because PACF of the AR(p) model becomes 0 for $k > p$.

—- To select the order q of MA(q) model, we use the sample ACF function. We test the hypothesis

$H_0: \rho_k = 0$ against alternative $H_1: \rho_k \neq 0$

at lags $k = 1, 2, ...$ at significance level 5%, where ρ_k is ACF function.

Rule: ACF ρ_k is significant at lag k at 5% significance level, if $|\hat{\rho}_k| > 2/\sqrt{N}$, where N is the number of observations.

If $|\hat{\rho}_k| \leq 2/\sqrt{N}$, then ACF at lag k is not significantly different from 0.

We select for q the largest lag k at which the ACF is significant.

(ii) We have $2/\sqrt{N} = 2/\sqrt{400} = 0.1$. The PACF is significant only at lag 1. Hence we would fit AR(1) model.

The ACF shows significant correlation at the lags 1 to 10. Hence we would fit MA(10) model.

From the two models AR(1) and MA(10) we select a simpler model AR(1) with smaller number of parameters which should to be fitted to the data.

(iii) According to (ii), we can fit AR(1) model

$$X_t = \phi_0 + \phi X_{t-1} + \varepsilon_t$$

where ε_t is white noise.

(iv) This AR(1) model would fit the data if residuals $\hat{\varepsilon}_t = X_t - \widehat{\phi} X_{t-1}$ are uncorrelated. We could use the correlogram of residuals to test whether residuals are uncorrelated.