MTH6113 Specimen Paper 3 Suggested Solutions

1. An investor has a utility from wealth described by: $U(w)=\sqrt{w}$ where $w>0$ is his/her wealth.
   a) Show that this investor prefers more to less and he/she is risk averse.

   Answer
   More is better is satisfied as the utility function is an increasing function of wealth:
   $U'(w)=\frac{1}{2\sqrt{w}}>0$
   Risk aversion satisfied as well as the utility function is a concave function in wealth:
   $U''(w)=-\frac{1}{4}w^{-3/2}<0$

   b) What are this investor's Arrow Pratt measures of risk aversion? Are these measures decreasing or increasing in wealth? Explain what these results convey.

$$ARA=-\frac{U''(w)}{U'(w)}=\frac{1}{2w}$$
$$RRA=-w\frac{U''(w)}{U'(w)}=\frac{1}{2}$$
$$\frac{\partial ARA}{\partial w}=-\frac{1}{2w^2}<0$$
$$\frac{\partial RRA}{\partial w}=0$$

   ARA decreasing in wealth -as the investor's wealth increases the absolute risk aversion decrease. This implies that this investor will invest more in risky assets in absolute value when their wealth increases.
   RRA neither decreasing nor increasing but constant: This implies that this investor will keep the proportion of risky assets out of their total wealth constant when their wealth increases.

   c) Initially the investor has a wealth of £400. He also has a lottery ticket that will be worth £1,200 with probability 0.5 and £0 with probability 0.5. What is his/her expected utility?

   Answer

   The investor faces a random event $X=\{1{,}200;0\}$with probabilities $\{0.5;0.5\}$
   His expected utility is:
$$EU(w_0+X)=0.5\sqrt{400+1{,}200}+0.5\sqrt{400}=0.5\times 40+0.5\times 20=30$$

d) Someone offers to buy this lottery ticket. What is the lowest price at which the investor will part with the lottery ticket?

Answer

If someone is buying the lottery for price P, the investor faces no uncertainty and hence his utility is $U(w_0 + P) = \sqrt{400 + P}$.

Hence the investor is indifferent between selling the lottery or not if: $U(w_0 + P) = \sqrt{400 + P} = 30 = EU(w_0 + X)$

Hence $400 + P = 900$, and $P = 500$

e) The investor considers investing in the two risky assets A and B with cumulative probability distribution functions:

$$F_A(w) = w$$

$$F_B(w) = w^{1/3} + w$$

i) Find the conditions for which these are cdfs well defined?

The conditions are:
$0 \leq F_A(w) = w \leq 1$
and $0 \leq F_B(w) = w^{1/3} + w \leq 1$
The last condition is equivalent with:

$$w^{\frac{1}{3}} \leq 1 - w$$

$$w \leq (1 - w)^3$$

$$w \leq 1 - 3w + 3w^2 - w^3$$

$$1 - 4w + 3w^2 - w^3 \geq 0$$

So, $0 \leq w \leq 0.31767$

For exam purposes just stating the conditions is enough.

ii) Under what conditions on w is A preferred to B on the basis of first-order stochastic dominance?

Answer:

A first order stochastically dominates B if

$F_A(w) \leq F_B(w)$ for all w and

$F_A(w) < F_B(w)$ for some w

In our problem this means

$w \leq w^{1/3} + w$ for all w and

$w < w^{1/3} + w$ for some w

Which implies $0 \leq w^{1/3}$ for all w and $0 < w^{1/3}$ for some w.

Thus the values for which these are satisfied are: $w \geq 0$

iii) Under what conditions on w is A preferred to B on the basis of second-order stochastic dominance?

<u>Answer</u>

A second order stochastically dominates B if

$$\int_0^w F_A(x)dx \leq \int_0^w F_B(x)dx \text{ for all } w \text{ and}$$

$$\int_0^w F_A(x)dx < \int_0^w F_B(x)dx \text{ for some } w$$

This implies:

$$\int_0^w xdx \leq \int_0^w (x + x^{1/3})dx \text{ or} \int_0^w x^{1/3}dx \geq 0 \text{ or} \left[\frac{3x^{4/3}}{4}\right]_0^w \geq 0$$

Hence,

$$\frac{3w^{4/3}}{4} \geq 0 \text{ for all values of } w \text{ and}$$

$$\frac{3w^{4/3}}{4} > 0 \text{ for some values of } w$$

Thus the values for which these are satisfied are: $w \geq 0$

2. An investor has access only to two stocks General Motors (GM) and Fiat Chrysler Automobiles (FCAU). The expected annual return on GM's shares is 10%, and the expected annual return on FCAU's shares is 7%. These stocks trade on NYSE. Today's price per share of GM opened at $43.40 and of FCAU at $24.50. The rates of return from these two companies' shares have a correlation coefficient of 0.8. The standard deviation of the rates of return on GM's shares is 0.04 and the standard deviation of the return on FCAU's shares is 0.08. The investor prefers more to less and can short sell both assets.

a. The investor buys 100 shares of GM and 10 shares of FCAU at the open price.

i) What is the market price of this portfolio?

Answer:

$$P = 100 \times \$43.40 + 10 \times \$24.50 = \$4{,}340 + \$245 = 4{,}585$$

ii) What are the weights of GM and FCAU in this portfolio?

Answer:

$$w_{GM} = \frac{\$4{,}340}{\$4{,}585} = 0.9466$$

$$w_{FCAU} = \frac{\$245}{\$4{,}585} = 0.0533$$

b. The investor plans to invest in an efficient portfolio. Explain what the investor understands by efficient portfolio.

Answer:

A portfolio is efficient if investor cannot find a better one.

Better is in terms of - lower variance with same/higher expected return or - higher expected return with same/lower variance.

c. What is the minimum global variance portfolio (V) for this investor? Interpret the weights on GM and FCAU.

$$w_{GM} = \frac{\sigma^2{}_{FCAU} - \sigma_{GM,FCAU}}{\sigma^2{}_{GM} - 2\sigma_{GM,FCAU} + \sigma^2{}_{FCAU}} = \frac{(0.08)^2 - 0.8 \times 0.04 \times 0.08}{(0.04)^2 - 2 \times 0.8 \times 0.04 \times 0.08 + (0.08)^2} = 1.33$$

$$w_{FCAU} = 1 - w_{GM} = -0.33$$

The investor short sell FCAU and takes a long position in GM.

d. What is the expected value and variance of the portfolio found at point (c)?

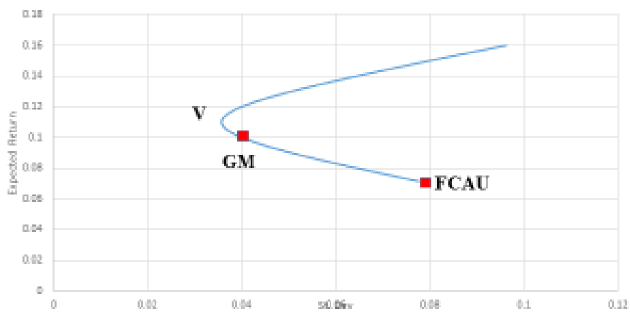
Answer

$$V_P = (1.33)^2 \times (0.04)^2 + (-0.33)^2 \times (0.08)^2 + 2 \times 1.33 \times (-0.33) \times 0.8 \times 0.08 \times 0.04 = 0.00128$$

$$E_P = 1.33 \times 0.1 - 0.33 \times 0.07 = 0.11$$

e. Draw the Mean Variance Frontier for this investor. Make sure you identify on the diagram the GM and FCAU stocks as well as the minimum variance portfolio V. Based on your diagram can you invest only on one of the stocks efficiently?

Answer:



Note that the expected return of the minimum variance portfolio is higher than that of GM and FCAU independently.
The efficient frontier is to the right and above V and both GM and FCAU are not on it.

f. Assume now that GM and FCAU are perfectly positively correlated. Can you offer a security with no risk in this economy? If yes, what are the weights of GM and FCAU in this new security?

Answer:
Yes: The easiest way to find the weights of this risk free portfolio is to realise that the standard deviation of GM is half the standard deviation of FCAU. Hence

$w_{GM} = 2$

$w_{FCAU} = -1$

Of course we could substitute the new correlation coefficient into minimum variance formula and we will get the same answer.

3. An investor believes that the upcoming referendum on the independence of one of the regions of the country results will play a role in the success of his business. There are only two possibilities, the region becomes independent or not. Recent polls have predicted that the electorate is split at 50% between the options.

The investor believes that its venture percentage annual returns, R, in the post-referendum scenario will have the following distribution: $R \sim Uniform(-2,2)$ if the L Party wins and $R \sim Uniform(-2,6)$ if the R Party wins. Note that annual return is a continuous random variable.

a) Calculate the conditional expected returns and variances of returns for the company, separately for the two possible alternative post-election scenarios.

Answer:

Note that for $R \sim Uniform(a, b)$

$$E(R) = \frac{a+b}{2}$$

$$Var(R) = \frac{(b-a)^2}{12}$$

If the L Party wins:

$$E(R) = \frac{2-2}{2} = 0$$

$$Var(R) = \frac{(2+2)^2}{12} = 1.33$$

If the R Party wins:

$$E(R) = \frac{-2+6}{2} = 2$$

$$Var(R) = \frac{(8)^2}{12} = 5.33$$

b) Calculate the unconditional expected returns and unconditional variance of returns for the company.

Answer:

Now we have to keep into account that $E(R)$ and $Var(R)$ are random variable with outcomes depending on the results of the election. The following table summarizes these two random variables:

	L wins with 0.5 probability	R wins with 0.5 probability
$E(R/Party)$	0	2
$Var(R/Party)$	1.33	5.33

Using the Law of iterated Expectations:

$$E(R) = E(E\left(\frac{R}{Party}\right)) = 0.5 \times 0 + 0.5 \times 2 = 1$$

We also know that:

$$Var(R) = Var(\left(E(R/Party)\right) + E(Var(R/Party))$$

$$Var(\left(E(R/Party)\right) = 0.5(0-1)^2 + 0.5(2-1)^2 = 1$$

$$E(Var(R/Party)) = 0.5 \times 1.33 + 0.5 \times 5.33 = 3.33$$

Hence, $Var(R) = 4.33$

c) Calculate the probability that the company will register negative returns.

Answer:

$$Prob(R < 0) = 0.5 \times \frac{2}{4} + 0.5 \times \frac{2}{8} = 0.375$$

d) Calculate the Value-at-Risk over a period of one year for the investment return R for the following probabilities of ruin equal to 0.375 and 0.1.

Answer:

Value at risk: For $Prob(R < L) = q$, $VaR(q) = -L$
For $q = 0.375$ from the part c) $VaR(0.375) = 0$

For, $q = 0.1$, $Prob(R < L) = 0.5 \times \frac{L+2}{4} + 0.5 \times \frac{L+2}{8} = 0.1$ or

$\frac{3}{8} + \frac{3L}{16} = 0.1$, or $L = -1.4667$.

Hence $VaR(0.1) = 1.4667$

e) Based on the different measures of risks and returns calculated above, summarise the company's outlook in the year following the election.

Answer:
i) The company can expect a higher expected return if the R party wins.
ii) However, the standard deviation of returns will be lower if the L party wins.
iii) Probability of negative returns is 0.375 irrespective of the winning party.
iv) The VaR increases, as the probability of ruin falls.

4. Suppose the annual rate of return on short-term government securities (risk-free) is 3%. Suppose asset A has a beta of 2 and an expected annual return of 15%.

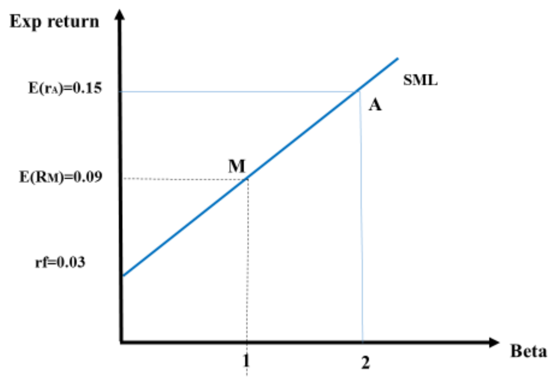
a) What is the expected return on the market according to CAPM.

Answer:

$$E(R_A) = r_f + \beta_A (E_M - r_f)$$

Thus: $E_M = \dfrac{E(R_A) - r_f}{\beta_A} + r_f = \dfrac{0.15 - 0.03}{2} + 0.03 = 0.09$ or 9%

b) Draw a diagram showing the security market line, the risk free rate, the expected annual return of the market and the annual return of asset A.



c) Calculate the expected annual return on an asset B with a beta of 0.7.

Answer:

$$E(R_B) = r_f + \beta_B (E_M - r_f) = 0.03 + 0.7 \times (0.09 - 0.03) = 0.03 + 0.07 \times 0.06 = 0.072$$

d) Suppose you bought asset B at £10 and sold it after one year for £12. Calculate the realized annual return on asset B.

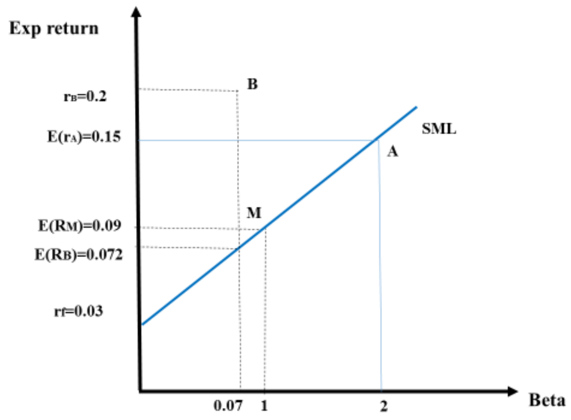
Answer: $R_B = \dfrac{120 - 100}{100} = 0.2$

e) Determine whether asset B is overpriced or underpriced by the market.

Answer:

Asset B has a higher return than the one determined through CAPM, which means that the market does not price correctly this asset. Asset B is underpriced by the market.

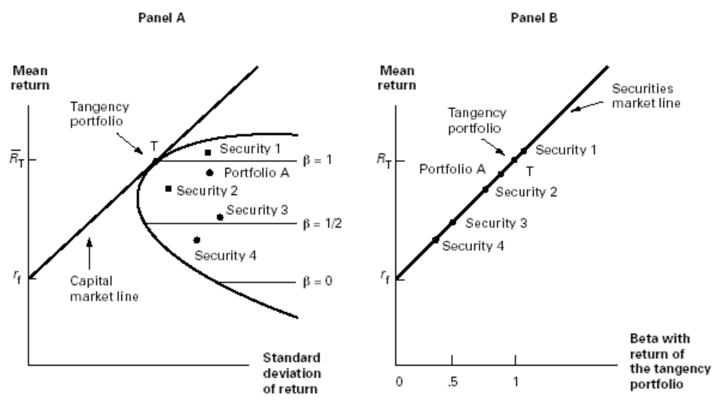
The alpha of stock B is: $\alpha_B = 0.2 - 0.072 = 0.128 > 0$



f) Explain the difference between security market line and capital market line.

CML graphs risk premiums of efficient portfolios as a function of portfolio standard deviation
Standard deviation is a valid measure of risk for efficiently diversified portfolios that are candidates for an investors' overall portfolio.

SML graphs individual asset risk premiums as a function of asset risk, where the appropriate risk measure is the contribution of that asset to the total portfolio risk - the beta



5.

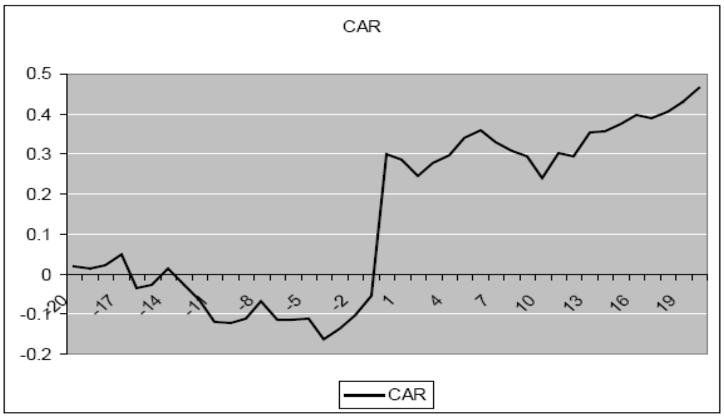
The market of a small developing nation is currently semi-strong form efficient. The government announces that, in order to make the market strong form efficient, it is passing a law forbidding employees of companies from transacting in shares of their own companies. Defining semi-strong and strong forms of efficient market hypothesis, discuss if the proposed law would succeed.

<u>Answer</u>

The semi-strong form of the EMH states that market prices already contain all publicly available information. Hence it is not possible to consistently outperform the market unless the someone has inside information.
The strong form of EMH states that market prices incorporate all information, both publicly available and also that available only to insiders. Examples of insiders who do not work for a company include: auditors, specialist advisers (banks, lawyers etc), large customers. So the law, forbidding only employees of companies from transacting

in shares of their own companies, is unlikely to succeed in making the market conform strong-form efficiency.

b) On October 24, 2004, after the close of trading, MEE (Massey Energy, a mining company) announced its third quarter earnings. On October 25 the stock reported a return of 37%. The graph below represents the daily Cumulative Abnormal Return for MEE for a 41-trading day window cantered around the announcement day (day 0). Is the evidence in this event study consistent with the efficient market hypothesis?



<u>Solution</u>

What we know:

A market is efficient with respect to a given information set Ω if no agent can make economic profit through the use of a trading rule based on Ω.

Efficient Market Hypothesis (EMH) states that stock prices already reflect all available information, hence changes in prices should be unpredictable (random)

EMH has three different versions based on definition of "all available information:"

- Weak-form hypothesis
- Semi-strong form hypothesis
- Strong form hypothesis

The question provides CAR: cumulative abnormal returns before and after announcement.

We are looking at available trading data, so we should analyse first whether the event shows consistency with the weak form of market efficiency.

Further we look at semi-strong of form market efficiency. As earnings announcements reflect the financial health of a firm, we would expect stock prices to rise upon the announcement of better-than-expected earnings (good news) and fall if earnings are below expectations (bad news.)

If market is semi-strong efficient then it is also weak efficient.

Notice that CAR shows no increasing pattern before the event. There is a large jump on the announcement day (this is consistent with semi-strong form market efficiency).

There is a slight decreasing pattern before the announcement. Perhaps investors are anticipating bad news. We cannot interpret this as inside information as the trend is opposite the actual announcement. So there is no evidence consistent with strong form of market consistency.

After the event, CAR shows no consistent pattern for the next few days (again, this is consistent with semi strong market efficiency). However, after day 10 CAR seems to show an upward drift. This is not consistent with semi-strong market efficiency (CAR should be flat after the event as the prices adjust instantenously). Is this due to the announcement or due to something else?

The empirical evidence from this event study shows that investors were positively surprised.

There is some evidence that seems to contradict market efficiency, but we need to study many more companies to draw any conclusion.