Q1 (a) Target: $M_S(t)$

(8) $M_S(t) = M_N(\ln M_X(t))$

$M_X(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2} = e^{20t + \frac{1}{2} \times 6^2 t^2} = e^{20t + 18t^2}$

Given

$M_N(t) = e^{\lambda(e^t - 1)} = e^{100(e^t - 1)}$

Given

$M_S(t) = M_N(\ln M_X(t)) = e^{100(e^{\ln M_X(t)} - 1)} = e^{100(M_X(t) - 1)}$

$= e^{100(e^{20t+18t^2} - 1)}$

(b) Target: $E(S)$, $Var(S)$ — Conclusion for Compound Poisson Distribution

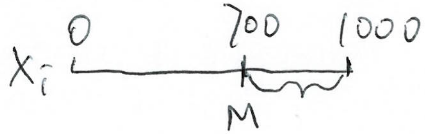
(7)

$E(S) = \lambda m_1 = 100 \times E(X) = 100 \times 20 = 2000$

$Var(S) = \lambda \underset{\substack{\| \\ E(X^2)}}{m_2} = 100 \times (\sigma^2 + \mu^2) = 100 \times (6^2 + 20^2) = 43600$

Q2 (a) Y_i : individual claim payments of insurers

$$m_1 = E(Y_i) = \int_0^M x\, f(x)\, dx + M\, P(X_i > M)$$



$$= \int_0^{700} x \frac{1}{1000-0}\, dx + 700 \times 0.3$$

$$\doteq 455$$

$$E(S_I) = \lambda E(Y_i) = 20 \times 455 = 9100$$

compound Poisson

$$m_2 = E(Y_i^2) = \int_0^M x^2 f(x)\, dx + M^2 P(X_i > M)$$

$$= \int_0^{700} x^2 \cdot \frac{1}{1000}\, dx + 700^2 \times 0.3$$

$$= 261{,}333.$$

$$Var(S_I) = \lambda E(Y_i^2) = 20 \times 261{,}333 = 5{,}226{,}660$$

$X_i \sim U(0, 1000)$

Q2 (b) $E(S_R) = E(S) - E(S_I) = \lambda E(X_i) - \underset{(a)}{E(S_I)} = 20 \times 500 - 9100 = 900$

$Var(S_R) \neq Var(S) - E(S_I)$

$E(Z_i^2) = \int_M^{1000} (x-M)^2 f(x)\, dx = \int_0^{1000-M} y^2\, 0.001\, dy = 9000$

set $y = x - M$

$Var(S_R) = \lambda E(Z_i^2) = 20 \times 9000 = 180{,}000$

(c) $Var(S) = \lambda E(X_i^2)$

$E(X_i^2) = \int_0^{1000} x^2 \cdot f(x)\, dx = \int_0^{1000} x^2 \cdot \frac{1}{1000}\, dx = \frac{10000}{3}$

$Var(S) = 20 \times \frac{10000}{3} = 6{,}666{,}666.66$

Q3. (a) $C(u,v)$ gives the probability that r.v. 1 is in the bottom uth percentile, and r.v. 2 is in the bottom vth percentile.

(b) $\lambda_L = \lim_{u\to 0^+} \frac{C[u,u]}{u}$ $\qquad C(u,v) = (u^{-\alpha} + v^{-\alpha} - 1)^{-\frac{1}{\alpha}}$ $\quad \alpha > 0$

$$C(u,u) = (2u^{-\alpha} - 1)^{-\frac{1}{\alpha}}$$

$$\lambda_L = \lim_{u\to 0^+} \frac{(2u^{-\alpha} - 1)^{-\frac{1}{\alpha}}}{u} = \lim_{u\to 0^+} \left(2 - \frac{1}{u^{-\alpha}}\right)^{-\frac{1}{\alpha}} = \lim_{u\to 0^+} (2 - u^{\alpha})^{-\frac{1}{\alpha}}$$

$$\overset{\alpha>0}{=} 2^{-\frac{1}{\alpha}}$$

(c) $P(X \le 40, Y \le 40) = C(u,v)$ $\quad$ where $u = P(X\le 40) = 0.16$, $v = P(Y \le 40) = 0.25$

$$= (u^{-\alpha} + v^{-\alpha} - 1)^{-\frac{1}{\alpha}}$$

$$= (0.16^{-0.5} + 0.25^{-0.5} - 1)^{-\frac{1}{0.5}} = 0.0816$$

(e)

$\psi^{[-1]}$

$$\psi(0) = \lim_{t\to 0} -\ln\left(\frac{e^{-\alpha t}-1}{e^{\alpha}-1}\right) = \cancel{\lim_{t\to 0} -\ln 0} = \infty$$

$$\therefore \boxed{\psi^{[-1]} = \psi^{(-1)}}$$

$$\boxed{y = \psi^{(-1)}(x)} \Rightarrow \psi(y) = x \qquad -\ln\left(\frac{e^{-\alpha y}-1}{e^{-\alpha}-1}\right) = x$$

$$y = -\frac{1}{\alpha}\ln\left[1+(e^{-\alpha}-1)e^{-x}\right]$$

$$\psi^{[-1]}(x) = -\frac{1}{\alpha}\ln\left[1+(e^{-\alpha}-1)e^{-x}\right]$$

$$C(u,v,w) = \psi^{[-1]}\left(\underbrace{\psi(u)+\psi(v)+\psi(w)}\right)$$

$$= -\frac{1}{\alpha}\ln\left[1+(e^{-\alpha}-1)e^{-\left[-\ln\left(\frac{e^{-\alpha u}-1}{e^{-\alpha}-1}\right)-\ln\left(\frac{e^{-\alpha v}-1}{e^{-\alpha}-1}\right)-\ln\left(\frac{e^{-\alpha w}-1}{e^{-\alpha}-1}\right)\right]}\right]$$

$$= \ldots$$

Q4. (a)

$$M_X(t) = E(e^{tX}) = \int_0^{\infty} \boxed{e^{tX}} \underset{0.02x\, e^{-x}}{f(x)}\, dx = \int_0^{\infty} 0.02\,x\, \boxed{e^{(t-1)x}}\, dx$$

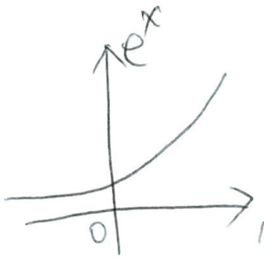
Integration by parts (mentioned in CW in tutorial)

$$\begin{cases} u = 0.02x \\ \dfrac{dv}{dx} = e^{(t-1)x} \end{cases} \qquad \frac{du}{dx} = 0.02 \qquad v = \int e^{(t-1)x}\, dx = \frac{e^{(t-1)x}}{(t-1)}$$

$$\cancel{\frac{e^{(t-1)x}}{(t-1)} \cdot (t-1) = e^{(t-1)x}}$$

$$M_X(t) = [uv]\Big|_0^{\infty} - \int_0^{\infty} v \frac{du}{dx}\, dx$$

$$= \left[\frac{0.02x\, e^{\overset{<0}{(t-1)}x}}{\underset{<0}{t-1}}\right]_0^{\infty} - \underset{\text{w.r.t } x}{\int_0^{\infty}} \frac{e^{(t-1)x}}{t-1} \times 0.02\, dx$$



$t<1$

$$= 0 - 0 - \left[\frac{0.02\, e^{(t-1)x}}{(t-1)^2}\right]_0^{\infty} = \frac{0.02}{(t-1)^2}$$

(b) $\underbrace{M_X(R)}_{(a)} = 1 + \frac{CR}{\lambda} = 1 + \frac{(1+\theta)E(X)\lambda R}{\lambda} = 1 + 1.3 \times \underbrace{E(X)}\cdot R$

! $\underbrace{E(X) = M_X'(0)} = \frac{d}{dt}\left[\frac{0.02}{(t-1)^2}\right]_{t=0} = \left.\frac{-2\times 0.02}{(t-0.01)^3}\right|_{t=0} = \underline{0.04}$

$\frac{0.02}{(R-1)^2} = 1 + 1.3 \times 0.04R$

$0.08\cancel{6} \sim \underline{0.087}$

$L = \frac{0.02}{(R-1)^2} - 1 \cancel{+} 1.3 \times 0.04R$

$L(0.086) \begin{smallmatrix} >0 \\ <0 \end{smallmatrix}$

$L(0.087) \begin{smallmatrix} <0 \\ >0 \end{smallmatrix}$

① monotonic

② Signs $\neq$