~~Week 12~~

Week 12 Revision

1. Collective Risk Models	Week 1-4	1 Q	
2. Extreme value theory	Week 5	1 Q	
3. Copula	Week 6	1 Q	
4. Ruin theory	Week 8-9	1 Q	
5. Run-off triangle	Week 10-11	0 Q	only in assessed coursework Excel

1. Collective risk models

1.1 X: loss distribution

MGF $M_X(t) = E(e^{tX})$ uniquely determines the distribution

Calculation Eg. Slide 9 week 1 MGF $X \sim$ Uniform

Conclusion: k^{th} moment derivide from MGF

$$M_X(t) = \sum_{k=0}^{\infty} E(X^k) \boxed{\frac{t^k}{k!}}$$

$$E(X^k) = \frac{d^k}{dt^k} M_X(t)\Big|_{t=0}$$

proof!

Common statistical distributions

	E(X)	Var(X)	MGF
Exponetial			Slide 14 W1
Gamma			slide 17 W1
			slide 19-20 W1
Normal			

$2\lambda X \sim \chi^2_{2\alpha}$

Log Normal

Pareto

Burr

Weibull

Calculation of $E(X)$, $Var(X)$, MGF , X No memorize

Estimate

① The method of Moments

~~E(X)~~ population moments = Sample moments

$$E(X) = \frac{1}{n}\sum_{i=0}^{n} x_i$$

Var

skew

depends on the number of unknown parameters

Eg. Slide 37 W1

② MLE

$$I(\theta) = \ln(L(\theta)) = \sum_{i=1}^{n} \ln p(X = x_i | \theta)$$

$$\frac{d}{d\theta} I(\hat{\theta}) = 0$$

Step 1: write down $\ln(L(\theta))$

Step 2: Take natural log

Step 3: Max $\ln(L(\theta))$

Step 4: Solve $\hat{\theta}$

Don't forget: second order derivatives

$$\frac{d^2}{d\theta^2} I(\hat{\theta}) < 0 \rightarrow \text{max}, \quad \text{not min}$$

Eg. Slide 50-52 Exponetial

slide 53-~~56~~ 59 Gamma, Normal, LnNormal, Pareto

~~Slide~~ Weibull, Burr.

③ The method of percentile

percentile of the population = percentile of the sample

depends on ~~on~~ the num of parameters

E.g. Slide 60

Goodness of fit

CW

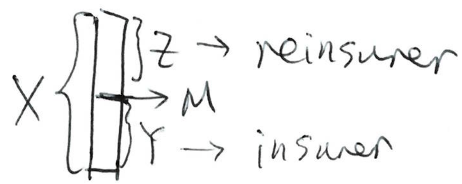
Week 2

Reinsurance

Def

proportional reinsurance

XOL



$Y = \{$

$Z = \{$

Eg
Slide 9 – 10 $E(X) = \int_0^\infty x f(x)\,dx$

$$E(Y) = \int_0^M x \cdot f(x)\,dx + M P(X>M), \quad E(Z) = \int_M^\infty (x-M) f(x)\,dx$$

MGF

$$M_Y(t) = E(e^{tY}) = \int_0^M e^{tx} f(x)\,dx + e^{tM} P(X>M)$$

Slide 11 $g(w) = \dfrac{f(w+M)}{1-F(M)}, \quad w>0$

Eg XOL Re Slide 13 W2

Proportional Re

$$\begin{cases} Y = \alpha X \\ Z = (1-\alpha) X \end{cases}$$

Eg. Slide 15 – 17

Calculation tricks × memorize Slide 18 – 20

Useful integral formulae: ✓ Calculation ✓ Apply

Infletion

$$\begin{cases} Y = kX & kX \le M \\ Y = M & kX > M \end{cases}$$

not proportional

Sample is consored ~~Eg Reinsurer~~

$\to$ Reinsurer Truncated

M

$\to$ insurer Censored

MLE: slide 24 $L_1(\theta) \times L_2(\theta)$

$\uparrow$ Complete data $\uparrow$ cencored data

censored data + MLE

Reinsurance

Slide 27 - 41 read by yourself, good for interview

Week 3 $X \to S$

$$S = \sum_{i=1}^{N} X_i \qquad X_i, \quad N, \quad r.v.s$$

~~Nomoth~~

Moments of S

$$E(S) = E\left[E(S|N)\right]$$

$$Var(S) = E\left[Var(S|N)\right] + Var\left[E(S|N)\right]$$

× proof ✓ apply

$$E(S) = E(N)\,E(X)$$

✓ proof ✓ apply it calculation Slide 12

$$Var(S) = E(N)\,Var(X) + Var(N)\left[E(X)\right]^2$$

$$M_S(t) = \cancel{E\left[E(e^{tS}|N\right.}\ M_N(\ln M_X(t))$$

Slide 15 - 17

The compound Poisson distribution $\quad N \sim \text{Poisson}(\lambda)$

$E(N) = Var(N) = \lambda \rightarrow$ memorise

$$\left.\begin{aligned} E(S) &= \lambda m_1 \\ Var(S) &= \lambda m_2 \\ Skew(S) &= \lambda m_3 \end{aligned}\right\}$$

✓ memorise ✓ proof ✓ apply to calculation

Sum of Compound Poisson distribution Slide 22 Eg Slide 23

$A = S_1 + S_2 + \cdots + S_n \qquad S_i \sim$ Compound Poisson λ_i

✓ understand

✓ simple calculation

$$\Lambda = \sum_{i=1}^{n} \lambda_i$$

$$F(x) = \frac{1}{\Lambda} \sum_{i=1}^{n} \lambda_i \, F_i(x)$$

The compound Binomial distribution $\quad N \sim Bin(n, p)$

$E(S) = ?$

$Var(S) = ?$

$M_S(t) = ?$

✓ Calculation slide 25

✓ application

Eg. Slide 27–29

Week 4

Reinsurance + S

Examples Slide 5 proportional Reins + S

slide 8 XOL Reins + S

Skip Slide 17 - 21 x detail ✓ read

Slide 22 Exam style question for Week 1-4

Variability of hetero / homo portfolio parameter : r.v.

Difference between the 2

X Calculation

✓ Example

2. Extreme value Theory   Week 5

CW   Worksheet

GEV + GPD

Def ← Slide 5   Slide 3 Q1 "extreme event"

motivation ← slide 3

GEV − Def   understand   max block

slide 8   X memorize, provided

$H(x) = \{$ ✓ Calculation

apply context

↓

$\alpha, \beta, \gamma \xrightarrow{\text{impact}} H(x)$ memorize:

	γ	real cases	upper bound	tail
Fréchet				
Weibull				
Gumbel	$=0$			

GEV distribution

Comment on the suitablity of the distribution: slide 14 + 10 + 11

GPD

Def ← understand Threshold u

$X-u \mid X>u \sim$ CDF slide 17

$G(x) = \{$ slide 19

β, γ

Measures of tail weight

1) The existence of moments → light tail slide 22 - 23

2) Limiting density ratios → memorise PDF

$$\lim_{x\to\infty} \frac{f_{X_1}(x)}{f_{X_2}(x)} \quad \begin{array}{ll} \to 0 & X_1 \text{ is lighter} \\ \to \infty & X_2 \text{ is lighter} \end{array}$$

✓ 3) Hazard rate

$$h(x) = \frac{f(x)}{1-F(x)}$$

✓ 4) Mean residual life

✓ Calculation

✓ compare tail weight of 2 distribution

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3. Copula

Def ← slide 9

function: input output slide 10

Eg. Slide 11

motivation

λ_u, λ_L slide 7, 8, 13, 15

Sklar's theorem ← understand existence of Copula, converse is also true

Fundamental Copulas
- Independence - product $P(AB) = P(A) \cdot P(B)$
- perfect positive interdependence - co-monotonic
  min
- perfect negative interdependence - counter-monotonic
  max

✓ understand : product, min, max

✓ examples Fundamental Copula

Explicit Copulas — Archimedean Copula
- Gumbel Copula
- Clayton Copula
- Frank Copula

big question!

Eg. Slide 24

$$\psi^{[-1]}(x) = \psi^{(-1)}(x) \qquad \psi(0) = \infty$$

Implicit Copulas
- Gaussian Copula
- Student's t Copula

Ruin theory

Def $\quad U(t) = U + ct - S(t) \qquad < 0$

$S \to S(t)$ ~~waiting time~~ ~ ~~Exponential~~

~~Poisson process~~

$\psi(u)$, $\psi(u, t)$, $\psi_h(u, t)$ $\leftarrow$ Def both { math / in words }

Comparisons ψ Slide 10–12

understand ~~Co~~ economically

Poisson process Def λ λh Slide 14

✓ understand def

✓ Proof

waiting time ~ Exponential Slide 15

✓ Apply calculation Time between claims timeline slide 16

$$P(N(t)=0) = e^{-\lambda t}$$

Example. slide 17 – 18

Compound Poisson process

$N \to N(t)$

$\lambda \to \lambda t$

Lunderg's inequality $\psi(u) \le e^{-Ru}$

R: adjustment coefficient $\psi(u) \approx e^{-Ru}$

R↑, risk↓

$\underline{\lambda M_X(R) - \lambda - cR = 0}$, $c = (1+\theta)\lambda m_1$, $\underline{M_X(R) = 1 + (1+\theta) m_1 R}$

E.g. Slide 33 W8

35

37 range different signs

Upper and lower bound of R Slide 40 - 45

Taylor's series

✗ ~~memo~~ memorize ✓ proof

Slide 45

Impact of parameters on ψ

u, c, θ, λ

Standardisation of unit

Poisson	1	1 pound = 100 pence
$E(X)$	1	

Eg. Slide 15

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5. Run-off triangles