

Useful Formulae

$$\begin{aligned}
A_x &= E(v^{K_x+1}) = \sum_{k=0}^{\infty} v^{k+1} {}_k|q_x \\
{}_nE_x &= E(v^n 1_{\{T_x > n\}}) = v^n {}_np_x = v^n \frac{l_{x+n}}{l_x} \\
A_{x:\overline{n}|}^1 &= E(v^{K_x+1} 1_{\{K_x+1 \leq n\}}) = A_x - v^n E_x A_{x+n} \\
A_{x:\overline{n}|} &= E(v^{\min(K_x+1, n)}) = A_{x:\overline{n}|}^1 + {}_nE_x \\
\ddot{a}_x &= E\left(\ddot{a}_{\overline{K_x+1}|}\right) = \sum_{k=0}^{\infty} v^k {}_kp_x = \sum_{k=0}^{\infty} \ddot{a}_{\overline{k+1}|} {}_k|q_x \\
\ddot{a}_x^{(m)} &= E\left(\ddot{a}_{\overline{K_x^{(m)} + \frac{1}{m}}|}\right) = \ddot{a}_x - \frac{m-1}{2m} \\
\ddot{a}_{x:\overline{n}|} &= E\left(\ddot{a}_{\overline{\min(K_x, n)}|}\right) = \ddot{a}_x - v^n {}_np_x \ddot{a}_{x+n} \\
A_x &= 1 - d\ddot{a}_x \\
A_{x:\overline{n}|} &= 1 - d\ddot{a}_{x:\overline{n}|}
\end{aligned}$$

T_x and T_y independent for the following formulae:

$$\begin{aligned}
{}_tP_{xy} &= {}_tP_x \times {}_tP_y \\
{}_tq_{xy} &= {}_tq_x + {}_tq_y - {}_tq_x {}_tq_y \\
{}_tP_{\overline{xy}} &= {}_tP_x + {}_tP_y - {}_tP_{xy} \\
\mu_{x+t:y+t} &= \mu_{x+t} + \mu_{y+t} \\
{}_k|q_{xy} &= {}_kP_{xy} q_{x+k:y+k} \\
{}_k|q_{\overline{xy}} &= {}_k|q_x + {}_k|q_y - {}_k|q_{xy} \\
{}_nq_{xy}^1 &= \int_0^n {}_tP_{xy} \mu_{x+t} dt \\
{}_nq_{xy}^1 &= \int_0^n {}_tP_{xy} \mu_{y+t} dt \\
\bar{A}_{xy}^1 &= \int_0^{\infty} v^t {}_tP_{xy} \mu_{x+t} dt \\
A_{xy}^1 &= \sum_0^{\infty} v^{t+1} {}_tP_{xy} q_{x+t:y+t}^1 \\
A_{xy}^2 &= A_y - A_{xy}^1 \\
\bar{A}_{xy}^2 &= \int_0^{\infty} v^t \bar{A}_{y+t} {}_tP_{xy} \mu_{x+t} dt \\
a_{x|y} &= a_y - a_{xy} = \frac{A_{xy} - A_y}{d}
\end{aligned}$$

$$\begin{aligned} \bar{a}_{x|y} &= \bar{a}_y - \bar{a}_{xy} = \frac{\bar{A}_{xy} - \bar{A}_y}{\delta} \\ {}_t p_x^{\bar{ii}} &= \exp\left(-\int_0^t \sum_{j=0, j \neq i}^m \mu_{x+s}^{ij} ds\right) \\ {}_{t+h} p_x^{ij} &= {}_t p_x^{ij} {}_h p_{x+t}^{jj} + \sum_{k=0, k \neq j}^m {}_t p_x^{ik} {}_h p_{x+t}^{kj} \\ \bar{a}_x^{ij} &= \int_0^\infty e^{-\delta t} {}_t p_x^{ij} dt \\ \bar{a}_{x:\bar{n}}^{ij} &= \bar{a}_x^{ij} - e^{-\delta n} \sum_{k=0}^m {}_n p_x^{ik} \bar{a}_{x+n}^{kj} \\ \bar{A}_x^{ij} &= \int_0^\infty \sum_{k=0, k \neq j}^\infty e^{-\delta t} {}_t p_x^{ik} \mu_{x+t}^{kj} dt \\ \bar{A}_{x:\bar{n}}^{ij} &= \bar{A}_x^{ij} - e^{-\delta n} \sum_{k=0}^m {}_n p_x^{ik} \bar{A}_{x+n}^{kj} \end{aligned}$$

The next pages contain extracts from the AM92 Mortality Tables

AM92

x	$l_{[x]}$	$l_{[x-1]+1}$	l_x	x
17	9 997.809 1		10 000.000 0	17
18	9 991.890 4	9 993.540 0	9 994.000 0	18
19	9 986.035 1	9 987.633 8	9 988.063 6	19
20	9 980.243 2	9 981.791 1	9 982.200 6	20
21	9 974.504 6	9 976.001 6	9 976.390 9	21
22	9 968.839 1	9 970.265 4	9 970.634 6	22
23	9 963.196 7	9 964.582 4	9 964.931 3	23
24	9 957.577 5	9 958.922 5	9 959.261 3	24
25	9 951.991 3	9 953.285 8	9 953.614 4	25
26	9 946.398 2	9 947.662 2	9 947.980 7	26
27	9 940.798 4	9 942.021 8	9 942.340 2	27
28	9 935.181 8	9 936.354 9	9 936.673 0	28
29	9 929.508 8	9 930.661 3	9 930.969 4	29
30	9 923.749 7	9 924.891 6	9 925.209 4	30
31	9 917.914 5	9 919.026 0	9 919.353 5	31
32	9 911.953 8	9 913.054 7	9 913.382 1	32
33	9 905.828 2	9 906.928 5	9 907.265 5	33
34	9 899.498 4	9 900.607 8	9 900.964 5	34
35	9 892.915 1	9 894.053 6	9 894.429 9	35
36	9 886.039 5	9 887.206 9	9 887.612 6	36
37	9 878.812 8	9 880.028 8	9 880.454 0	37
38	9 871.166 5	9 872.450 8	9 872.895 4	38
39	9 863.022 7	9 864.404 7	9 864.868 8	39
40	9 854.303 6	9 855.793 1	9 856.286 3	40
41	9 844.902 5	9 846.538 4	9 847.051 0	41
42	9 834.703 0	9 836.524 5	9 837.066 1	42
43	9 823.599 4	9 825.635 4	9 826.206 0	43
44	9 811.447 3	9 813.746 3	9 814.335 9	44
45	9 798.083 7	9 800.693 9	9 801.312 3	45
46	9 783.337 1	9 786.316 2	9 786.953 4	46
47	9 766.998 3	9 770.423 1	9 771.078 9	47
48	9 748.860 3	9 752.787 4	9 753.471 4	48
49	9 728.649 9	9 733.193 8	9 733.886 5	49
50	9 706.097 7	9 711.352 4	9 712.072 8	50
51	9 680.899 0	9 686.966 9	9 687.714 9	51
52	9 652.696 5	9 659.707 5	9 660.502 1	52
53	9 621.100 6	9 629.211 5	9 630.052 2	53
54	9 585.691 6	9 595.056 3	9 595.971 5	54
55	9 545.992 9	9 556.800 3	9 557.817 9	55
56	9 501.483 9	9 513.937 5	9 515.104 0	56
57	9 451.593 8	9 465.929 3	9 467.290 6	57
58	9 395.697 1	9 412.171 2	9 413.800 4	58
59	9 333.128 4	9 352.016 5	9 354.004 0	59
60	9 263.142 2	9 284.764 1	9 287.216 4	60
61	9 184.968 7	9 209.656 8	9 212.714 3	61
62	9 097.740 5	9 125.881 8	9 129.717 0	62
63	9 000.588 4	9 032.564 2	9 037.397 3	63
64	8 892.574 1	8 928.817 7	8 934.877 1	64

AM92

x	$l_{[x]}$	$l_{[x-1]+1}$	l_x	x
65	8 772.735 9	8 813.688 1	8 821.261 2	65
66	8 640.048 1	8 686.201 6	8 695.619 9	66
67	8 493.518 7	8 545.353 2	8 557.011 8	67
68	8 332.139 6	8 390.161 1	8 404.491 6	68
69	8 154.931 8	8 219.639 0	8 237.132 9	69
70	7 960.977 6	8 032.860 6	8 054.054 4	70
71	7 749.465 9	7 828.968 6	7 854.450 8	71
72	7 519.702 7	7 607.240 0	7 637.620 8	72
73	7 271.146 1	7 367.082 8	7 403.008 4	73
74	7 003.521 6	7 108.105 2	7 150.240 1	74
75	6 716.823 1	6 830.184 4	6 879.167 3	75
76	6 411.345 9	6 533.500 8	6 589.925 8	76
77	6 087.808 4	6 218.575 9	6 282.980 3	77
78	5 747.362 4	5 886.362 8	5 959.168 0	78
79	5 391.640 0	5 538.279 1	5 619.757 7	79
80	5 022.793 1	5 176.222 4	5 266.460 4	80
81	4 643.512 9	4 802.629 0	4 901.478 9	81
82	4 257.005 6	4 420.452 5	4 527.496 0	82
83	3 866.988 4	4 033.146 7	4 147.670 8	83
84	3 477.592 9	3 644.632 7	3 765.599 8	84
85	3 093.286 3	3 259.186 2	3 385.247 9	85
86	2 718.712 8	2 881.346 7	3 010.839 5	86
87	2 358.529 9	2 515.731 0	2 646.741 6	87
88	2 017.229 8	2 166.880 5	2 297.297 6	88
89	1 698.908 9	1 839.045 8	1 966.649 9	89
90	1 407.055 0	1 535.980 1	1 658.554 5	90
91		1 260.735 4	1 376.190 6	91
92			1 121.988 9	92
93			897.502 5	93
94			703.324 2	94
95			539.064 3	95
96			403.402 3	96
97			294.206 1	97
98			208.706 0	98
99			143.712 0	99
100			95.847 6	100
101			61.773 3	101
102			38.379 6	102
103			22.928 4	103
104			13.135 9	104
105			7.196 8	105
106			3.759 6	106
107			1.866 9	107
108			0.878 4	108
109			0.390 3	109
110			0.163 2	110
111			0.064 0	111
112			0.023 4	112
113			0.008 0	113
114			0.002 5	114
115			0.000 7	115
116			0.000 2	116
117			0.000 0	117
118			0.000 0	118
119			0.000 0	119
120			0.000 0	120

AM92

4%

x	$\ddot{a}_{[x]}$	$A_{[x]}$	${}^2A_{[x]}$	\ddot{a}_x	A_x	2A_x	x
17	23.372	0.101 08	0.016 96	23.367	0.101 27	0.017 16	17
18	23.280	0.104 60	0.017 78	23.276	0.104 78	0.017 97	18
19	23.185	0.108 27	0.018 67	23.180	0.108 44	0.018 85	19
20	23.086	0.112 10	0.019 64	23.081	0.112 26	0.019 82	20
21	22.982	0.116 08	0.020 70	22.978	0.116 24	0.020 86	21
22	22.874	0.120 23	0.021 84	22.870	0.120 38	0.022 00	22
23	22.762	0.124 55	0.023 08	22.758	0.124 69	0.023 24	23
24	22.645	0.129 05	0.024 43	22.641	0.129 19	0.024 58	24
25	22.523	0.133 73	0.025 89	22.520	0.133 86	0.026 03	25
26	22.396	0.138 60	0.027 47	22.393	0.138 73	0.027 61	26
27	22.265	0.143 67	0.029 17	22.261	0.143 79	0.029 31	27
28	22.128	0.148 94	0.031 02	22.124	0.149 06	0.031 15	28
29	21.985	0.154 42	0.033 01	21.982	0.154 54	0.033 14	29
30	21.837	0.160 11	0.035 15	21.834	0.160 23	0.035 28	30
31	21.683	0.166 03	0.037 47	21.680	0.166 15	0.037 59	31
32	21.523	0.172 18	0.039 96	21.520	0.172 30	0.040 08	32
33	21.357	0.178 57	0.042 64	21.354	0.178 68	0.042 76	33
34	21.185	0.185 20	0.045 52	21.182	0.185 31	0.045 65	34
35	21.006	0.192 07	0.048 61	21.003	0.192 19	0.048 74	35
36	20.821	0.199 21	0.051 93	20.818	0.199 33	0.052 07	36
37	20.628	0.206 60	0.055 49	20.625	0.206 72	0.055 63	37
38	20.429	0.214 26	0.059 30	20.426	0.214 39	0.059 45	38
39	20.223	0.222 20	0.063 38	20.219	0.222 34	0.063 54	39
40	20.009	0.230 41	0.067 75	20.005	0.230 56	0.067 92	40
41	19.788	0.238 91	0.072 41	19.784	0.239 07	0.072 59	41
42	19.560	0.247 70	0.077 38	19.555	0.247 87	0.077 58	42
43	19.324	0.256 78	0.082 67	19.319	0.256 96	0.082 89	43
44	19.080	0.266 15	0.088 32	19.075	0.266 36	0.088 56	44
45	18.829	0.275 83	0.094 31	18.823	0.276 05	0.094 58	45
46	18.569	0.285 80	0.100 68	18.563	0.286 05	0.100 98	46
47	18.302	0.296 07	0.107 44	18.295	0.296 35	0.107 78	47
48	18.027	0.306 64	0.114 60	18.019	0.306 95	0.114 98	48
49	17.745	0.317 52	0.122 17	17.736	0.317 86	0.122 60	49
50	17.454	0.328 68	0.130 17	17.444	0.329 07	0.130 65	50
51	17.156	0.340 14	0.138 61	17.145	0.340 58	0.139 15	51
52	16.851	0.351 89	0.147 49	16.838	0.352 38	0.148 11	52
53	16.538	0.363 92	0.156 84	16.524	0.364 48	0.157 55	53
54	16.218	0.376 23	0.166 65	16.202	0.376 85	0.167 45	54
55	15.891	0.388 79	0.176 93	15.873	0.389 50	0.177 85	55
56	15.558	0.401 61	0.187 69	15.537	0.402 40	0.188 74	56
57	15.219	0.414 66	0.198 93	15.195	0.415 56	0.200 12	57
58	14.874	0.427 94	0.210 64	14.847	0.428 96	0.212 00	58
59	14.523	0.441 43	0.222 82	14.493	0.442 58	0.224 37	59
60	14.167	0.455 10	0.235 47	14.134	0.456 40	0.237 23	60
61	13.808	0.468 94	0.248 57	13.769	0.470 41	0.250 58	61
62	13.444	0.482 92	0.262 11	13.401	0.484 58	0.264 40	62
63	13.077	0.497 03	0.276 08	13.029	0.498 90	0.278 68	63
64	12.708	0.511 23	0.290 46	12.653	0.513 33	0.293 40	64

Note. ${}^2A_{[x]} = A_{[x]}$ at 8.16% and ${}^2A_x = A_x$ at 8.16%.

AM92

4%

x	$\ddot{a}_{[x]}$	$A_{[x]}$	${}^2A_{[x]}$	\ddot{a}_x	A_x	2A_x	x
65	12.337	0.525 50	0.305 22	12.276	0.527 86	0.308 55	65
66	11.965	0.539 81	0.320 33	11.896	0.542 46	0.324 10	66
67	11.592	0.554 14	0.335 78	11.515	0.557 10	0.340 03	67
68	11.221	0.568 44	0.351 51	11.135	0.571 75	0.356 30	68
69	10.850	0.582 70	0.367 51	10.754	0.586 38	0.372 89	69
70	10.481	0.596 87	0.383 72	10.375	0.600 97	0.389 75	70
71	10.116	0.610 93	0.400 12	9.998	0.615 48	0.406 86	71
72	9.754	0.624 85	0.416 65	9.623	0.629 88	0.424 16	72
73	9.396	0.638 60	0.433 27	9.252	0.644 14	0.441 62	73
74	9.044	0.652 14	0.449 93	8.886	0.658 24	0.459 19	74
75	8.698	0.665 45	0.466 59	8.524	0.672 14	0.476 83	75
76	8.359	0.678 51	0.483 20	8.169	0.685 81	0.494 48	76
77	8.027	0.691 27	0.499 71	7.820	0.699 24	0.512 10	77
78	7.703	0.703 73	0.516 09	7.478	0.712 38	0.529 65	78
79	7.388	0.715 85	0.532 27	7.144	0.725 23	0.547 07	79
80	7.082	0.727 62	0.548 22	6.818	0.737 75	0.564 32	80
81	6.785	0.739 03	0.563 90	6.502	0.749 93	0.581 36	81
82	6.499	0.750 05	0.579 27	6.194	0.761 75	0.598 14	82
83	6.222	0.760 68	0.594 30	5.897	0.773 19	0.614 61	83
84	5.957	0.770 90	0.608 95	5.610	0.784 25	0.630 75	84
85	5.701	0.780 72	0.623 20	5.333	0.794 90	0.646 52	85
86	5.457	0.790 12	0.637 01	5.066	0.805 14	0.661 88	86
87	5.223	0.799 11	0.650 38	4.811	0.814 98	0.676 80	87
88	5.000	0.807 69	0.663 29	4.566	0.824 39	0.691 27	88
89	4.788	0.815 85	0.675 73	4.332	0.833 38	0.705 25	89
90	4.586	0.823 62	0.687 68	4.109	0.841 96	0.718 74	90
91				3.897	0.850 12	0.731 72	91
92				3.695	0.857 87	0.744 17	92
93				3.504	0.865 22	0.756 09	93
94				3.323	0.872 18	0.767 48	94
95				3.153	0.878 75	0.778 34	95
96				2.992	0.884 94	0.788 67	96
97				2.840	0.890 77	0.798 47	97
98				2.698	0.896 25	0.807 76	98
99				2.564	0.901 39	0.816 54	99
100				2.439	0.906 21	0.824 83	100
101				2.321	0.910 71	0.832 63	101
102				2.212	0.914 92	0.839 97	102
103				2.110	0.918 85	0.846 86	103
104				2.015	0.922 51	0.853 31	104
105				1.926	0.925 91	0.859 34	105
106				1.844	0.929 07	0.864 98	106
107				1.768	0.932 01	0.870 23	107
108				1.697	0.934 72	0.875 12	108
109				1.632	0.937 24	0.879 66	109
110				1.571	0.939 56	0.883 87	110
111				1.516	0.941 70	0.887 77	111
112				1.464	0.943 67	0.891 37	112
113				1.417	0.945 49	0.894 69	113
114				1.374	0.947 15	0.897 75	114
115				1.334	0.948 68	0.900 56	115
116				1.298	0.950 08	0.903 15	116
117				1.264	0.951 39	0.905 57	117
118				1.229	0.952 73	0.908 04	118
119				1.176	0.954 78	0.911 81	119
120				1.000	0.961 54	0.924 56	120

Note. ${}^2A_{[x]} = A_{[x]}$ at 8.16% and ${}^2A_x = A_x$ at 8.16%.

End of Appendix.

