

MTH6113 Mathematical Tools for Asset Management

Specimen Paper 2

Time: 3 hours

1. A shepherd has a utility of wealth $u(w) = w^{\frac{2}{3}}$ for $w > 0$ and $u(0) = 0$.
- Show that the shepherd is:
 - non-satiated, and
 - risk averse.
 - Calculate the coefficients of risk aversion and explain what they convey.

The shepherd begins every year with a herd of sheep with value £100,000, but over the year, she loses some sheep due to wolf attacks. If the winters are hard, she loses 0.25% of her herd, and if the winters are mild, she loses 0.05% of her herd. Every year there are 50-50 chances of either hard or mild winter. The shepherd is considering buying a sheepdog that will protect the herd in case of wolf attack. A fellow shepherd tells her that there are three type of breeds that are the best for this task: Border Collie, Groenendael and Berger de Brie. These breeds are guaranteed to protect fully against any wolf attack.

The shepherd has done some research about the three breeds and she summarised her findings in the Table below:

	Border Collie	Groenendael	Berger de Brie
Current price per dog	£1,200	£1,300	£1,400
Average working life before retirement	10 years	13 years	7 years

If the shepherd buys the dog there is an agreement in place with the sellers to spread the cost over the average working life of the dog. Note that this is a zero-interest agreement between buyers and sellers.

- Calculate the average expected annual return for the three dogs.
- Calculate the standard deviation of the returns for these dogs.

The shepherd decides which dog to buy based on a performance measure r , which is defined as the ratio of expected annual return to standard deviation of returns.

- Provide a rationale behind the investor's choice of the performance measure r .
- Find which dog the shepherd buys based on the performance measure.

2. The shepherd from Question 1 purchased a sheepdog and now she decides to insure her sheepdog against theft and physical damage. One of the most common medical problems of sheepdogs is retinal atrophy and the shepherd is looking for an insurance that covers this medical problem. Her estimate of the amount of possible losses from owning a sheepdog and the associated probabilities are as given in the table below:

Type of loss and value	Probability
Theft: £1,000	0.2
Retinal atrophy: £800	0.2
Other medical problems: £200	0.3

An insurance company with a capital of £100,000 appraises the potential of losses and their associated probabilities in the same way as the dog owner. The utility function of the insurance company is $u(c) = 10,000 + 0.5 c$, where $c > 0$ is the insurer's capital.

- Calculate the minimum premium the insurer is willing to offer for a policy that insures the above losses.
 - Calculate the maximum price that the dog owner is prepared to pay in order to insure her dog.
 - Determine whether the insurance company and the dog owner can agree on a policy that is advantageous to both parties.
3. The forecast for the economic outlook is that, the economy will be either in recession with probability 0.25, or experience normal growth with probability 0.75. There are two stocks available to trade in this economy (stock SeaLab and stock BinTech). The stocks' annual returns in each of the states of the world are according to the following table:

Stocks	Recession	Normal Growth
SeaLab	8%	12%
BinTech	16%	10%

- Show whether you can rank the stocks by the criterion of first order stochastic dominance. If yes, which stock first order stochastically dominates the other?
- Show whether you can rank the stocks by the criterion of second order stochastic dominance. If yes, which stock second order stochastically dominates the other?
- Calculate the expected annual return of each stock.
- Calculate the annual variance and standard deviation of each stock.
- Calculate the covariance and correlation coefficient between the two stocks.

- f) An investor plans to set up a risk-free portfolio with these two stocks. Determine what weights he needs to put on each of the stocks such that he achieves this goal.
 - g) Assuming that the CAPM holds calculate the composition of the market portfolio with expected return 10% per annum.
 - h) Calculate the beta of each security, under the assumption that the risk-free rate of interest is 1% per annum and interpret your results.
4. Which forms of market efficiency are satisfied and/or violated in the following situations:
- i) The host of TV show E-Investments gives stock recommendations every day and insists following these recommendations will beat the market.
 - ii) A pharmaceutical company announces that it has discovered a vaccine for violet fever. Its share price rises immediately by 12%.
 - iii) If a firm's stock price falls by more than 1% on any given day, the return is typically positive the following day.
5. The trustees of a charity that supports research into violet fever, have approached a financial consultant to find out about behavioural finance and its relevance to the trustees' work.
- a. Briefly discuss what is meant by behavioural finance
 - b. State, giving reasons, two examples of behavioural finance themes that are relevant to the trustees of the charity.
 - c. Suggest reasons for the trustees' interest in learning about behavioural finance.