# MATH 5105 Differential and Integral Analysis
# Assignment 5

1. Let $f_n(x) = \frac{1+\cos^{2023}(nx)}{\sqrt{n}}$. Does $\{f_n\}$ converge uniformly to a function $f$ on $\mathbb{R}$?

2. (a) Show for all $x \in \mathbb{R}$, the sum $\sum_{n=1}^{\infty} \frac{1}{n^2} \cos\left(\frac{x}{n}\right)$ converges uniformly.
   (b) Show for all $x \in \mathbb{R}$, the sum $\sum_{n=1}^{\infty} \frac{1}{n^3} \sin\left(\frac{x}{n}\right)$ converges uniformly.

3. Does the pointwise limit function of the $\sum_{n=1}^{\infty} \frac{1}{n} \cos\left(\frac{x}{n}\right)$ exist as a function $f : \mathbb{R} \to \mathbb{R}$, i.e. for all $x \in \mathbb{R}$? If the function $f$ exists, decide whether or not the convergence is uniform.

4. Find a relationship between $\frac{1}{n^2} \cos\left(\frac{x}{n}\right)$ and $\frac{1}{n^3} \sin\left(\frac{x}{n}\right)$ and use this to show $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \sum_{n=1}^{\infty} \frac{1}{n^2} \cos\left(\frac{x}{n}\right)$$

   is differentiable.