Specimen Exam MTH6113

1. An investor has a utility from wealth described by: $U(w) = w$ where $w > 0$ is their wealth.
   a) Explain whether this investor prefers more to less and whether they are risk averse.

   [4 marks]

   Answer

   More is better is satisfied as the utility function is an increasing function of wealth: $U'(w) = 1 > 0$

   Risk aversion is not satisfied as the utility function is a linear function in wealth: $U''(w) = 0$ and hence the investor is risk neutral

   b) What are this investor's Arrow Pratt measures of risk aversion? Explain what these results convey.

   [6 marks]

   Answer

   The investor is not risk averse but risk neutral, so no calculation are needed in order to say that that ARA and RRA are 0. However, if the students use the formulae they should also get the same answer.

$$ARA = -\frac{U''(w)}{U'(w)} = 0$$

$$RRA = -w\frac{U''(w)}{U'(w)} = 0$$

   The degree of risk aversion is reflected in the concavity of the utility function and the investor attitude towards risk may change depending on the level of wealth. However the attitude towards risk of a risk neutral investor will not change with wealth.

   c) The investor is considering entering a business venture that involves a 20% probability of an income of £1,000 and 80% probability of an income of £200 depending on the state of the economy. The initial investment involves a payment of I_0. The investor's initial wealth is $w_0 = £1{,}000$. Using the definition of a "fair gamble", find the level of initial investment that guarantees this venture is a fair gamble for the investor.

   [5 marks]

   Answer: A gamble is fair if the expected wealth is equal with the initial wealth.

   In this case:

$$E(w) = -I_0 + 0.2 \times (w_0 + 1{,}000) + 0.8 \times (w_0 + 200) = -I_0 + w_0 + 360$$

   In order the venture to be a fair gamble: $E(w) = w_0$ and I_0 needs to be 360.

   d) Explain, giving reasons, whether the investor accepts the business venture at the investment level found in part c).

Answer:

When evaluating an investment, an investor compares the expected utility of wealth from taking a gamble with the sure utility of keeping the initial wealth and not entering the gamble: $E\big(v(w)\big) = U(w_0)$. The investor has a utility of wealth linear and hence he attaches the same utility to an incremental increase in wealth to an incremental decrease. Hence a risk neutral investor is indifferent about accepting or rejecting a fair bet as in expectation he will end up with the same utility as if not entering the bet.

[5 marks]
Total: [20 marks]

2. Rogers Ltd. is considering investing in two stocks, FTD and BLN. The expected annual return on FTD is 6% and the expected annual return on BLN is 14%. The rates of return from these two companies' shares have a correlation coefficient of $\rho = 0.03$. The standard deviation of the rates of return on FTD is 0.03 and the standard deviation of the return on BLN is 0.04. An investor prefers more to less and can short sell both assets (i.e. hold negative amounts of either asset).
   a) A portfolio P is formed using only FTD and BLN.
      i) Calculate the portfolio weights on FTD and BLN in order for P to be a global minimum variance portfolio.

      [4 marks]

      ii) Calculate the expected return and the standard deviation for this portfolio P.

      [4 marks]

Answer

$$w_{FTD} = \frac{\sigma_{BLN}^2 - \rho\sigma_{BLN}\sigma_{FTD}}{\sigma_{FTD}^2 + \sigma_{BLN}^2 - 2\rho\sigma_{BLN}\sigma_{FTD}} = \frac{0.0016 - 0.03 \times 0.03 \times 0.04}{0.0016 + 0.0009 - 2 \times 0.03 \times 0.03 \times 0.04}$$

$$w_{FTD} = 0.644$$

Hence,

$$w_{BLN} = 1 - 0.644 = 0.356$$
$$E(r_P) = 0.644 \times 0.06 + 0.356 \times 0.14 = 0.088$$

$$Var(r_P) = 0.644^2 \times 0.0009 + 0.356^2 \times 0.0016 + 2 \times 0.644 \times 0.356 \times 0.03 \times 0.03 \times 0.04 = 0.000593$$
$$\sigma_P = 0.0243$$

   b) Suppose the investor requires an expected return of 8% with the lowest possible variance from a portfolio Q formed using FTD and BLN shares. Calculate the required weights on FTD and BLN.

   [4 marks]

Answer:
Investment decision is:

$$min_{w_i} Var(r_Q) \text{ such that}$$

$$E\big(r_Q\big) = w_{FTD} \times 0.06 + w_{BLN} \times 0.14 = 0.08$$
$$w_{FTD} + w_{BLN} = 1$$

Set up the Lagrangian:

$$L(w_{FTD}, w_{BLN}, \lambda, \mu) = Var(r_Q) + \lambda(E(r_Q) - 0.08) + \mu(1 - w_{FTD} - w_{BLN})$$

which need to be minimized over $w_{FTD}, w_{BLN}, \lambda, \mu$ .This implies a system of 4 equations 4 unknowns. Since we are not interested in λ, μ the solution will be given by the constraints:

$$w_{FTD} \times 0.06 + w_{BLN} \times 0.14 = 0.08$$
$$w_{FTD} + w_{BLN} = 1$$

or

$$w_{FTD} \times 0.06 + (1 - w_{FTD}) \times 0.14 = 0.08$$

and

$$w_{FTD} = 0.75$$
$$w_{BLN} = 0.25$$

c) Assume now that the annual rates of return of FTD and BLN are in fact perfectly correlated with $\rho = -1$.

i) Can the investor build up a portfolio F with zero risk? If no, explain why. If yes, calculate this portfolio's weights on FTD and BLN.

[4 marks]

Answer

Yes, the weights of the risk free asset are calculated as:

$w_{FTD} = \frac{\sigma_{BLN}^2 + \sigma_{BLN}\sigma_{FTD}}{\sigma_{FTD}^2 + \sigma_{BLN}^2 + 2\sigma_{BLN}\sigma_{FTD}}$ if $\rho = -1$ and hence:

$$w_{FTD} = \frac{0.0016 + 0.03 \times 0.04}{0.0009 + 0.0016 + 2 \times 0.03 \times 0.04} = 0.57$$

$$w_{BLN} = 0.43$$

ii) Calculate the expected return of the risk free portfolio.

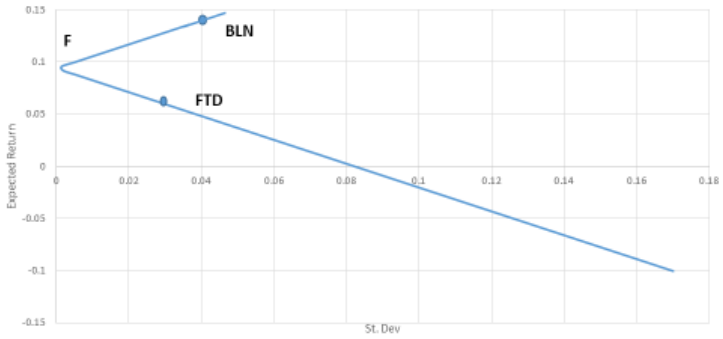
[2 mark]

Answer:

$$E(r_F) = 0.57 \times 0.06 + 0.43 \times 0.14 = 0.094$$

iii) Show and explain the efficient frontier using a standard "expected return – standard deviation" diagram that displays the annual returns of FTD, BLN, and F.

[4 marks]



iv) Describe how different risk averse investors find their optimum portfolios, using a diagram to support your answer.

[4 marks]

In an economy with a risk free asset and two assets, the efficient frontier will be fully described by the line that passes through the risk free asset (F) and the asset with higher expected return (BLN).

$$E(r_P) = r_F + \frac{E(r_{BLN}) - r_F}{\sigma_{BLN}}\sigma_P$$

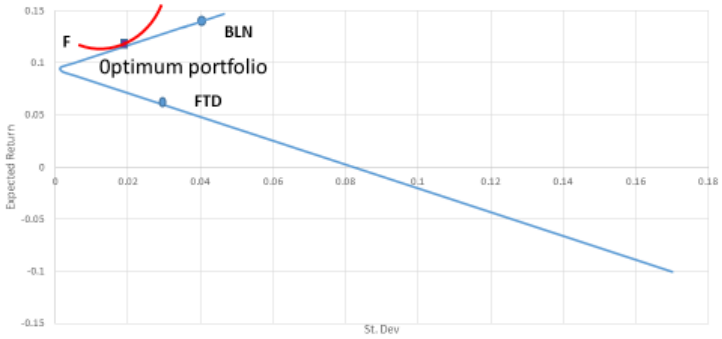
Substituting the risk free rate and the expected annual return and standard deviation of BLN we get:

$$E(r_P) = 0.094 + \frac{0.14 - 0.094}{0.04}\sigma_P$$

or:

$$E(r_P) = 0.094 + 1.15\,\sigma_P$$

We need to combine the investor preference for risk with the portfolio efficient frontier. The preference for risk is represented by investors indifference curves. The optimum can be found at the point that the indifference curve is tangent to the efficient frontier. It is important to state that different investors have difference preferences for risk and hence different indifference curves. Their optimum portfolios are not the same.



Total: [26 marks]

3. On January 1st, 2024 the 1-year Hungarian Government Bonds are offered at 1% yield. An investor is interested in investing in the Hungarian capital market. He has the following information about the expected annual returns, variances and correlations for two Hungarian cargo airlines (assuming these are the only stocks traded on this market): ASL Airlines and Fleet Air.

	Correlations with			
Stock	ASL Air	Fleet Air	Expected Annual Return	Variance
ASL Air	1	0.6	5%	0.16
Fleet Air	0.6	1	7%	0.09

a)

i) Do the 1-year Hungarian Government bonds bear any risk? Explain any assumptions you made in arriving at your answer.

[5 marks]

Answer

Risk free assets are assets that offer a sure return without any associated risks. The 1 year Hungarian Government bonds promise a sure return of 1% after a year. Hence they could be seen as risk free assets.

We have assumed that there is no political or economic risks that could make the Hungarian Government to not be able to fulfil its promise of payment of the announced yield at maturity. These risks should be non-diversifible or systematic risks.

Answers that argue that the Hungarian Bonds bear risk due to political uncertainty or economic instability should be marked based on the details of the argument.

ii) Find the annual risk free rate in this economy.

[2 mark]

Answer
Annual risk free rate is equal with the annual bond yield.

$$r_F = 0.01$$

If the students argued that the bonds bear a risk answer should be:

$$r_F = 0.01 - \lambda_l$$

Where λ_l accounts for the factor (risk) premium.

iii) Calculate the annual risk premium of ASL Air.

[3 mark]

Answer: The risk premium of a stock is defined as $E(R_j) - r_F$, hence ASL Air has a risk premium of 4% and Fleet Air a risk premium of 6%.

b) In order to decide his optimum portfolio, the investor needs to know the tangency portfolio in this market. Find the weights of the tangency portfolio.

[5 marks]

Answer:

Let the tangency portfolio K have weight w_A on ASL Air and w_F on Fleet Air, thus:

$$E(R_K) = w_A E_A + w_F E_F$$
$$w_A + w_F = 1$$

We also know the covariances of each of the stock with the tangency portfolio are:

$$\sigma_{AK} = w_A \sigma_A^2 + w_F \sigma_{AF}$$
and
$$\sigma_{FK} = w_A \sigma_{AF} + w_F \sigma_F^2$$

Where $\sigma_{AF} = \rho \sigma_A \sigma_F = 0.6 \times 0.4 \times 0.3 = 0.072$ is the covariance between the returns of ASL Air and Fleet Air.
Thus:

$$\sigma_{AK} = 0.16 w_A + 0.072 w_F$$
$$\sigma_{FK} = 0.072 w_A + 0.09 w_F$$

The β representation of the mean variance optimization problem is:

$$E(R_j) = r_F + \beta_j (E(R_K) - r_F)$$

$$\beta_j = \frac{\sigma_{jK}}{\sigma^2{}_K}$$

This means that $\frac{E(R_j) - r_F}{\sigma_{jK}} = \frac{E(R_K) - r_F}{\sigma^2{}_K}$ which means that ratio of risk premium of every stock and its covariance with the tangency portfolio is identical for every stock as $\frac{E(R_K) - r_K}{\sigma^2{}_K}$ is constant over all stocks, hence:

$$\frac{E(r_A) - r_A}{\sigma_{AK}} = \frac{E(r_F) - r_F}{\sigma_{FK}}$$

which becomes:

$$(0.16 w_A + 0.072 w_F) \times 0.06 = 0.04 \times (0.072 w_A + 0.09 w_F)$$

Knowing that $w_A + w_F = 1$ we can find w_A and w_F.

$$0.16 w_A + 0.072(1 - w_A) = \frac{2}{3} \times (0.072 w_A + 0.09(1 - w_A))$$

$$0.088 w_A + 0.072 = -0.012 w_A + 0.06$$

$$0.09 w_A = -0.012$$

$$w_A = -0.1333$$

$$w_F = 1.1333$$

c) Calculate the expected annual return and variance of the tangency portfolio.

[5 marks]

Answer:

$$E(R_K) = -0.1333 \times 0.05 + 1.1333 \times 0.07 = 0.07267$$

$$\sigma_K^2 = (-0.1333)^2 \times 0.16 + 1.1333^2 \times 0.09 - 2 \times 0.1333 \times 1.1333 \times 0.4 \times 0.3 = 0.082178$$

Total: [20 marks]

4.

a) An analyst has access to all the annual, monthly and daily returns of Coca Cola since it started trading on NYSE in 1919. Explain how the analyst would test whether the NYSE is efficient, by presenting any two statistical tests.

[6 marks]

A market is efficient with respect to a given information set **Ω** if no agent can make economic profit through the use of a trading rule based on **Ω**. By economic profit we understand the level of return after costs are adjusted appropriately for risk

Efficient Market Hypothesis (EMH) state that stock prices already reflect all available information, hence any changes in prices should be unpredictable (random).
EMH: if prices are determined rationally, then only new information will cause them to change. We cannot make excess returns based on a certain information set in an efficient market.

Excess returns are defined as $r_t^X = r_t - E(r_t)$ where r_t is the actual stock return at time *t* and $E(r_t)$ is the expected return motivated by a theoretical model (such as CAPM, ATP, etc), market model, or just a naïve model where the expected returns are equal with a constant.

The tests that can be used are:
<u>Covariance test:</u> Current and past asset returns should have no predictive power for future returns on that asset, hence one cannot form a trading rule based on current and historical returns of Coca Cola: $E\left(r_{t+1}|r_t, r_{t-1}, r_{t-2,\ldots.}\right) = 0$ which implies:

$$Cov(r_t, r_{t-s}) = 0 \ \ \forall s > 0$$

Use Coca Cola historical returns to test it.

Random Walk Model: Past price changes carry no information about current or future price changes.
A random walk model of Coca Cola price change can be written as:

$$P_t = P_{t-1} + \varepsilon_t$$

or $P_t - P_{t-1} = \varepsilon_t$ with

$$E(\varepsilon_t) = 0 \; \forall t > 1$$

and

$$Cov(\varepsilon_t, \varepsilon_s) = 0 \; \forall t \neq s$$

ε_t is new public information arriving at market during period *t*.

As Coca Cola price return is the first difference of the log Coca Cola price:

$$r_t = P_t - P_{t-1}$$

or

$$r_t = \varepsilon_t$$

Additional tests could be based on the fact that EHM is violated if any information available at time t or before allowed us to forecast returns
Regression tests such as:

$$r_{t+1} = \alpha + \beta X_t + \varepsilon_t$$

$$E(\varepsilon_t) = 0 \text{ and } Var(\varepsilon_t) = \sigma^2$$

X_t any possible forecasting variable for returns: calendar effects, certain accounting ratios, different trading strategies, etc.

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b) Define all the levels of market efficiency that you know.

[6 marks]

EMH has three different versions based on definition of all available information:

- Weak-form hypothesis: stock prices already reflect all information that can be derived by examining market trading data such as historical prices, volumes, etc.
- Semi-strong form hypothesis: stock prices. incorporate all publicly available information regarding the firms prospects (market trading data + future projects, earning forecasts)
- Strong form hypothesis: stock prices reflect all information relevant to the rm (including private information of company insiders + all publicly available information)

c) Which type of market efficiency did the analyst test in point (a)?

[6 marks]

The analyst tests at point (b) weak-form market efficiency as he examines historical market trading data.

d) A colleague of the analyst suggested that NYSE is not efficient as Coca Cola's returns can be forecasted based on the returns of Pepsi Cola. His argument is that, in the absence of arbitrage, all assets with identical factor exposures or risks earn the same return. Explain whether he is correct or not.

[4 marks]

Total: [20 marks]

Answer:

The colleague is making his argument based on arbitrage pricing theory which indeed states that in the absence of arbitrage, all assets with identical factor exposures or risks should earn the same return.

However, the mistake is making is that the efficient market hypothesis actually says that risk-adjusted returns should be unpredictable: $r_t^X = r_t - E(r_t)$.

The expected return $E(r_t)$ of Coca Cola is indeed the same as that of Pepsi if they face the same systematic risks or factors as per APT, but this does not imply that Coca Cola's risk adjusted returns $r_t^X = r_t - E(r_t)$ can be forecasted based on the historical returns of Pepsi.

5. You are working in the research department of an investment bank. A stock you have analysed has underperformed relative to its industry peers for the last three years. In a report for the front office you state that you are 90% sure that this stock will underperform in the coming year as well.

a) Discuss whether your advice could be biased, based on concepts learnt in behavioural finance.

[7 marks]

b) Before you submit your report, you learn of a rumour that the company might soon announce higher than expected earnings for the current year. However, you decide that this rumour is unfounded and do not incorporate it in your report. Describe a bias that could explain your decision.

[7 marks]

Total: [14 marks]

To be done after we learn behavioural finance

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